

Let P_{cd}^0 be the point such that lines P_{cd}^0C and AB are parallel (and PRATIO $P_{cd}^0 C A B 1$).

Let P_{ad}^1 be the point such that lines P_{ad}^1A and BC are parallel (and PRATIO $P_{ad}^1 A B C 1$).

$$\begin{aligned}
(3 \cdot S_{ABCD}) &= (13 \cdot S_{ABA_2}) && \text{by the statement (0)} \\
(3 \cdot S_{ABCD}) &= \left(13 \cdot \frac{((S_{ABB_1} \cdot S_{ABA_1}) + (-1 \cdot S_{A_1BB_1} \cdot S_{ABA_1}))}{S_{ABA_1B_1}} \right) && \begin{array}{l} \text{by Lemma} \\ 30 \text{ (point } A_2 \text{ eliminated)} \end{array} \quad (1) \\
(3 \cdot S_{ABCD}) &= \left(13 \cdot \frac{((S_{ABB_1} \cdot S_{ABA_1}) + (-1 \cdot S_{A_1BB_1} \cdot S_{ABA_1}))}{S_{ABA_1B_1}} \right)^2 && \begin{array}{l} \text{by Lemma} \\ 10 \text{ (equal)} \end{array} \quad (2) \\
(3 \cdot S_{ABCD}) &= \left(13 \cdot \frac{((S_{ABB_1} \cdot S_{ABA_1}) + (-1 \cdot 0))}{S_{ABA_1B_1}} \right) && \begin{array}{l} \text{multiplication} \\ \text{by 0} \end{array} \quad (3) \\
(3 \cdot S_{ABCD}) &= \left(13 \cdot \frac{((S_{ABB_1} \cdot S_{ABA_1}) + 0)}{S_{ABA_1B_1}} \right) && \begin{array}{l} \text{by multiplication} \\ \text{by 0} \end{array} \quad (4) \\
(3 \cdot S_{ABCD}) &= \left(13 \cdot \frac{(S_{ABB_1} \cdot S_{ABA_1})}{S_{ABA_1B_1}} \right) && \begin{array}{l} \text{by addition with} \\ 0 \end{array} \quad (5) \\
(3 \cdot S_{ABCD}) &= \frac{(13 \cdot (S_{ABB_1} \cdot S_{ABA_1}))}{S_{ABA_1B_1}} && \begin{array}{l} \text{by multiplication} \\ \text{of fractions} \end{array} \quad (6) \\
(3 \cdot S_{ABCD}) &= \frac{(13 \cdot ((S_{ABD} + (0.333333 \cdot (S_{ABD} + (-1 \cdot S_{ABD})))) \cdot S_{ABA_1}))}{S_{ABA_1B_1}} && \begin{array}{l} \text{by Lemma} \\ 29 \text{ (point } B_1 \text{ eliminated)} \end{array} \quad (7) \\
(3 \cdot S_{ABCD}) &= \frac{(13 \cdot ((S_{ABD} + (0.333333 \cdot (0 + (-1 \cdot S_{ABD})))) \cdot S_{ABA_1}))}{S_{ABA_1B_1}} && \begin{array}{l} \text{by Lemma} \\ 10 \text{ (equal)} \end{array} \quad (8) \\
(3 \cdot S_{ABCD}) &= \frac{(13 \cdot ((S_{ABD} + (0.333333 \cdot (-1 \cdot S_{ABD}))) \cdot S_{ABA_1}))}{S_{ABA_1B_1}} && \begin{array}{l} \text{by addition with} \\ 0 \end{array} \quad (9) \\
(3 \cdot S_{ABCD}) &= \frac{(13 \cdot ((S_{ABD} + (-0.333333 \cdot S_{ABD})) \cdot S_{ABA_1}))}{S_{ABA_1B_1}} && \begin{array}{l} \text{multiplication} \\ \text{of constants} \end{array} \quad (10) \\
(3 \cdot S_{ABCD}) &= \frac{(13 \cdot (((0.666667 \cdot S_{ABD}) + 0) \cdot S_{ABA_1}))}{S_{ABA_1B_1}} && \begin{array}{l} \text{by similar sum-} \\ \text{mands} \end{array} \quad (11) \\
(3 \cdot S_{ABCD}) &= \frac{(13 \cdot ((0.666667 \cdot S_{ABD}) \cdot S_{ABA_1}))}{S_{ABA_1B_1}} && \begin{array}{l} \text{by addition with} \\ 0 \end{array} \quad (12) \\
(3 \cdot S_{ABCD}) &= \frac{(13 \cdot (0.666667 \cdot (S_{ABD} \cdot S_{ABA_1})))}{S_{ABA_1B_1}} && \begin{array}{l} \text{by right associa-} \\ \text{tion} \end{array} \quad (13) \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot S_{ABA_1}))}{S_{ABA_1B_1}} && \begin{array}{l} \text{by multiplication} \\ \text{of constants} \end{array} \quad (14) \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot S_{ABA_1}))}{(S_{ABA_1D} + (0.333333 \cdot (S_{ABA_1A} + (-1 \cdot S_{ABA_1D}))))} && \begin{array}{l} \text{by Lemma} \\ 29 \text{ (point } B_1 \text{ eliminated)} \end{array} \quad (15)
\end{aligned}$$

$$\begin{aligned}
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot S_{ABA_1}))}{((S_{ABA_1} + S_{AA_1D}) + (0.333333 \cdot (S_{ABA_1A} + (-1 \cdot S_{ABA_1D}))))} \quad \text{by Definition 4 (16)} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot S_{ABA_1}))}{((S_{ABA_1} + S_{DAA_1}) + (0.333333 \cdot (S_{ABA_1A} + (-1 \cdot S_{ABA_1D}))))} \quad \text{by Lemma 1 (17)} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot S_{ABA_1}))}{((S_{ABA_1} + S_{DAA_1}) + (0.333333 \cdot ((S_{ABA_1} + S_{AA_1A}) + (-1 \cdot S_{ABA_1D}))))} \quad \text{by Definition 4 (18)} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot S_{ABA_1}))}{((S_{ABA_1} + S_{DAA_1}) + (0.333333 \cdot ((S_{ABA_1} + S_{AAA_1}) + (-1 \cdot S_{ABA_1D}))))} \quad \text{by Lemma 1 (19)} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot S_{ABA_1}))}{((S_{ABA_1} + S_{DAA_1}) + (0.333333 \cdot ((S_{ABA_1} + S_{AAA_1}) + (-1 \cdot (S_{ABA_1} + S_{AA_1D}))))} \quad \text{by Definition 4 (20)} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot S_{ABA_1}))}{((S_{ABA_1} + S_{DAA_1}) + (0.333333 \cdot ((S_{ABA_1} + S_{AAA_1}) + (-1 \cdot (S_{ABA_1} + S_{DAA_1}))))} \quad \text{by Lemma 1 (21)} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot S_{ABA_1}))}{((S_{ABA_1} + S_{DAA_1}) + ((0.333333 \cdot (S_{ABA_1} + S_{AAA_1})) + (0.333333 \cdot (-1 \cdot (S_{ABA_1} + S_{DAA_1}))))} \quad \text{by distribution of multiplication over addition (22)} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot S_{ABA_1}))}{((S_{ABA_1} + S_{DAA_1}) + ((0.333333 \cdot (S_{ABA_1} + S_{AAA_1})) + (-0.333333 \cdot (S_{ABA_1} + S_{DAA_1}))))} \quad \text{by multiplication of constants (23)} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot S_{ABA_1}))}{((S_{ABA_1} + S_{DAA_1}) + (((0.333333 \cdot S_{ABA_1}) + (0.333333 \cdot S_{AAA_1})) + (-0.333333 \cdot (S_{ABA_1} + S_{DAA_1}))))} \quad \text{by distribution of multiplication over addition (24)} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot S_{ABA_1}))}{(((1.333333 \cdot S_{ABA_1}) + S_{DAA_1}) + ((0.333333 \cdot S_{AAA_1}) + (-0.333333 \cdot (S_{ABA_1} + S_{DAA_1}))))} \quad \text{by similar summands (25)} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot S_{ABA_1}))}{(((1.333333 \cdot S_{ABA_1}) + S_{DAA_1}) + ((0.333333 \cdot S_{AAA_1}) + (-0.333333 \cdot (S_{ABA_1} + S_{DAA_1}))))} \quad \text{by addition with 0 (26)} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot S_{ABA_1}))}{(((1.333333 \cdot S_{ABA_1}) + S_{DAA_1}) + ((0.333333 \cdot S_{AAA_1}) + ((-0.333333 \cdot S_{ABA_1}) + (-0.333333 \cdot S_{DAA_1}))))} \quad \text{by distribution of multiplication over addition (27)} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot S_{ABA_1}))}{(((1 \cdot S_{ABA_1}) + S_{DAA_1}) + ((0.333333 \cdot S_{AAA_1}) + (0 + (-0.333333 \cdot S_{DAA_1}))))} \quad \text{by similar summands (28)} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot S_{ABA_1}))}{((S_{ABA_1} + S_{DAA_1}) + ((0.333333 \cdot S_{AAA_1}) + (0 + (-0.333333 \cdot S_{DAA_1}))))} \quad \text{by multiplication (29)} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot S_{ABA_1}))}{((S_{ABA_1} + S_{DAA_1}) + ((0.333333 \cdot S_{AAA_1}) + (-0.333333 \cdot S_{DAA_1})))} \quad \text{by addition with 0 (30)} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot S_{ABA_1}))}{((S_{ABA_1} + (0.666667 \cdot S_{DAA_1})) + ((0.333333 \cdot S_{AAA_1}) + 0))} \quad \text{by similar summands (31)} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot S_{ABA_1}))}{((S_{ABA_1} + (0.666667 \cdot S_{DAA_1})) + (0.333333 \cdot S_{AAA_1}))} \quad \text{by addition with 0 (32)}
\end{aligned}$$

$$\begin{aligned}
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot (S_{ABC} + (0.333333 \cdot (S_{ABD} + (-1 \cdot S_{ABC}))))))}{((S_{ABA_1} + (0.666667 \cdot S_{DAA_1})) + (0.333333 \cdot S_{AAA_1}))} && \begin{array}{l} \text{by Lemma 29} \\ \text{point A1 (33)} \\ \text{eliminated} \end{array} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot (S_{ABC} + (0.333333 \cdot (S_{ABD} + (-1 \cdot S_{ABC}))))))}{((S_{ABA_1} + (0.666667 \cdot S_{DAA_1})) + (0.333333 \cdot 0))} && \begin{array}{l} \text{by Lemma 34} \\ \text{equal} \end{array} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot (S_{ABC} + (0.333333 \cdot (S_{ABD} + (-1 \cdot S_{ABC}))))))}{((S_{ABA_1} + (0.666667 \cdot S_{DAA_1})) + 0)} && \begin{array}{l} \text{by multiplication (35)} \\ \text{eliminated} \end{array} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot (S_{ABD} \cdot (S_{ABC} + (0.333333 \cdot (S_{ABD} + (-1 \cdot S_{ABC}))))))}{(S_{ABA_1} + (0.666667 \cdot S_{DAA_1}))} && \begin{array}{l} \text{by addition with (36)} \\ \text{eliminated} \end{array} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot ((S_{ABD} \cdot S_{ABC}) + (S_{ABD} \cdot (0.333333 \cdot (S_{ABD} + (-1 \cdot S_{ABC}))))))}{(S_{ABA_1} + (0.666667 \cdot S_{DAA_1}))} && \begin{array}{l} \text{by distribution of multiplication (37)} \\ \text{over addition} \end{array} \\
(3 \cdot S_{ABCD}) &= \frac{(8.666671 \cdot ((S_{ABD} \cdot S_{ABC}) + (0.333333 \cdot (S_{ABD} \cdot (S_{ABD} + (-1 \cdot S_{ABC}))))))}{(S_{ABA_1} + (0.666667 \cdot S_{DAA_1}))} && \begin{array}{l} \text{by associativity and commutativity (38)} \\ \text{eliminated} \end{array} \\
(3 \cdot S_{ABCD}) &= \frac{((8.666671 \cdot (S_{ABD} \cdot S_{ABC})) + (8.666671 \cdot (0.333333 \cdot (S_{ABD} \cdot (S_{ABD} + (-1 \cdot S_{ABC}))))))}{(S_{ABA_1} + (0.666667 \cdot S_{DAA_1}))} && \begin{array}{l} \text{by distribution of multiplication (39)} \\ \text{over addition} \end{array} \\
(3 \cdot S_{ABCD}) &= \frac{((8.666671 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot (S_{ABD} + (-1 \cdot S_{ABC}))))))}{(S_{ABA_1} + (0.666667 \cdot S_{DAA_1}))} && \begin{array}{l} \text{by multiplication (40)} \\ \text{of constants} \end{array} \\
(3 \cdot S_{ABCD}) &= \frac{((8.666671 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot ((S_{ABD} \cdot S_{ABD}) + (S_{ABD} \cdot (-1 \cdot S_{ABC}))))))}{(S_{ABA_1} + (0.666667 \cdot S_{DAA_1}))} && \begin{array}{l} \text{by distribution of multiplication (41)} \\ \text{over addition} \end{array} \\
(3 \cdot S_{ABCD}) &= \frac{((8.666671 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot ((S_{ABD} \cdot S_{ABD}) + (-1 \cdot (S_{ABD} \cdot S_{ABC}))))))}{(S_{ABA_1} + (0.666667 \cdot S_{DAA_1}))} && \begin{array}{l} \text{by associativity and commutativity (42)} \\ \text{eliminated} \end{array} \\
(3 \cdot S_{ABCD}) &= \frac{((8.666671 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})) + (2.888887 \cdot (-1 \cdot (S_{ABD} \cdot S_{ABC}))))}{(S_{ABA_1} + (0.666667 \cdot S_{DAA_1}))} && \begin{array}{l} \text{by distribution of multiplication (43)} \\ \text{over addition} \end{array} \\
(3 \cdot S_{ABCD}) &= \frac{((8.666671 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})) + (-2.888887 \cdot (S_{ABD} \cdot S_{ABC})))}{(S_{ABA_1} + (0.666667 \cdot S_{DAA_1}))} && \begin{array}{l} \text{by multiplication (44)} \\ \text{of constants} \end{array} \\
(3 \cdot S_{ABCD}) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})) + 0))}{(S_{ABA_1} + (0.666667 \cdot S_{DAA_1}))} && \begin{array}{l} \text{by similar sum (45)} \\ \text{eliminated} \end{array} \\
(3 \cdot S_{ABCD}) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})))}{(S_{ABA_1} + (0.666667 \cdot S_{DAA_1}))} && \begin{array}{l} \text{by addition with (46)} \\ \text{eliminated} \end{array} \\
(3 \cdot S_{ABCD}) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})))}{((S_{ABC} + (0.333333 \cdot (S_{ABD} + (-1 \cdot S_{ABC})))) + (0.666667 \cdot S_{DAA_1}))} && \begin{array}{l} \text{by Lemma 29} \\ \text{point A1 (47)} \\ \text{eliminated} \end{array} \\
(3 \cdot S_{ABCD}) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})))}{((S_{ABC} + ((0.333333 \cdot S_{ABD}) + (0.333333 \cdot (-1 \cdot S_{ABC})))) + (0.666667 \cdot S_{DAA_1}))} && \begin{array}{l} \text{by distribution of multiplication (48)} \\ \text{over addition} \end{array}
\end{aligned}$$

$$\begin{aligned}
(3 \cdot S_{ABCD}) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})))}{((S_{ABC} + ((0.333333 \cdot S_{ABD}) + (-0.333333 \cdot S_{ABC}))) + (0.666667 \cdot S_{DAA_1}))} \quad \text{by multiplication (40)} \\
(3 \cdot S_{ABCD}) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})))}{(((0.666667 \cdot S_{ABC}) + ((0.333333 \cdot S_{ABD}) + 0)) + (0.666667 \cdot S_{DAA_1}))} \quad \text{by similar sum (50)} \\
(3 \cdot S_{ABCD}) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})))}{(((0.666667 \cdot S_{ABC}) + (0.333333 \cdot S_{ABD})) + (0.666667 \cdot S_{DAA_1}))} \quad \text{by addition with (51)} \\
(3 \cdot S_{ABCD}) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})))}{(((0.666667 \cdot S_{ABC}) + (0.333333 \cdot S_{ABD})) + (0.666667 \cdot (S_{DAC} + (0.333333 \cdot (S_{ACD} + (-0.333333 \cdot S_{ACD}))))))} \quad \begin{array}{l} \text{by Lemma 29 (point A)} \\ \text{eliminated} \end{array} \quad (52) \\
(3 \cdot S_{ABCD}) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})))}{(((0.666667 \cdot S_{ABC}) + (0.333333 \cdot S_{ABD})) + (0.666667 \cdot (S_{DAC} + (0.333333 \cdot (S_{ACD} + (-0.333333 \cdot S_{ACD}))))))} \quad \text{by Lemma 29 (point A)} \quad (53) \\
(3 \cdot S_{ABCD}) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})))}{(((0.666667 \cdot S_{ABC}) + (0.333333 \cdot S_{ABD})) + (0.666667 \cdot (S_{ACD} + (0.333333 \cdot (S_{ACD} + (-0.333333 \cdot S_{ACD}))))))} \quad \text{by Lemma 1} \quad (54) \\
(3 \cdot S_{ABCD}) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})))}{(((0.666667 \cdot S_{ABC}) + (0.333333 \cdot S_{ABD})) + (0.666667 \cdot (S_{ACD} + (0.333333 \cdot (S_{ACD} + (-0.333333 \cdot S_{ACD}))))))} \quad \text{by Lemma 1} \quad (55) \\
(3 \cdot S_{ABCD}) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})))}{(((0.666667 \cdot S_{ABC}) + (0.333333 \cdot S_{ABD})) + (0.666667 \cdot (S_{ACD} + (0.333333 \cdot (S_{ACD} + (-0.333333 \cdot S_{ACD}))))))} \quad \text{by addition with (56)} \quad (56) \\
(3 \cdot S_{ABCD}) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})))}{(((0.666667 \cdot S_{ABC}) + (0.333333 \cdot S_{ABD})) + (0.666667 \cdot (S_{ACD} + (-0.333333 \cdot S_{ACD}))))} \quad \text{by multiplication (57)} \quad (57) \\
(3 \cdot S_{ABCD}) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})))}{(((0.666667 \cdot S_{ABC}) + (0.333333 \cdot S_{ABD})) + (0.666667 \cdot ((0.666667 \cdot S_{ACD}) + 0)))} \quad \text{by similar sum (58)} \quad (58) \\
(3 \cdot S_{ABCD}) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})))}{(((0.666667 \cdot S_{ABC}) + (0.333333 \cdot S_{ABD})) + (0.666667 \cdot (0.666667 \cdot S_{ACD})))} \quad \text{by addition with (59)} \quad (59) \\
(3 \cdot S_{ABCD}) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})))}{(((0.666667 \cdot S_{ABC}) + (0.333333 \cdot S_{ABD})) + (0.444445 \cdot S_{ACD}))} \quad \text{by multiplication (60)} \quad (60) \\
\left(3 \cdot \frac{\left((S_{ACP_{cd}^0} \cdot S_{ABCP_{ad}^1}) + \left(-1 \cdot \left(S_{ad}^{CP_{cd}^1} \cdot S_{ad}^{ABCP_{ad}^1} \right) \right) \right)}{S_{ACP_{ad}^1} P_{cd}^0} \right) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})))}{(((0.666667 \cdot S_{ABC}) + (0.333333 \cdot S_{ABD})) + (0.444445 \cdot S_{ACD}))} \quad \begin{array}{l} \text{by Lemma 30} \\ \text{(point D eliminated)} \end{array} \quad (61) \\
\left(3 \cdot \frac{\left((S_{ACP_{cd}^0} \cdot S_{ABCP_{ad}^1}) + \left(-1 \cdot \left(S_{ad}^{CP_{cd}^1} \cdot S_{ad}^{ABCP_{ad}^1} \right) \right) \right)}{S_{ACP_{ad}^1} P_{cd}^0} \right) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})))}{(((0.666667 \cdot S_{ABC}) + (0.333333 \cdot S_{ABD})) + (0.444445 \cdot S_{ACD}))} \quad \text{by multiplication (62)} \quad (62) \\
\left(\left(3 \cdot (S_{ACP_{cd}^0} \cdot S_{ABCP_{ad}^1}) \right) + \left(3 \cdot \left(-1 \cdot (S_{ad}^{CP_{cd}^1} \cdot S_{ad}^{ABCP_{ad}^1}) \right) \right) \right) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})))}{(((0.666667 \cdot S_{ABC}) + (0.333333 \cdot S_{ABD})) + (0.444445 \cdot S_{ACD}))} \quad \begin{array}{l} \text{by distribution} \\ \text{of multiplication (63)} \\ \text{over addition} \end{array} \quad (63) \\
\left(\left(3 \cdot (S_{ACP_{cd}^0} \cdot S_{ABCP_{ad}^1}) \right) + \left(-3 \cdot (S_{ad}^{CP_{cd}^1} \cdot S_{ad}^{ABCP_{ad}^1}) \right) \right) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})))}{(((0.666667 \cdot S_{ABC}) + (0.333333 \cdot S_{ABD})) + (0.444445 \cdot S_{ACD}))} \quad \text{by multiplication (64)} \quad (64) \\
\left(\left(\left(3 \cdot (S_{ACP_{cd}^0} \cdot S_{ABCP_{ad}^1}) \right) + \left(-3 \cdot (S_{ad}^{CP_{cd}^1} \cdot S_{ad}^{ABCP_{ad}^1}) \right) \right) \right) &= \frac{((5.777784 \cdot (S_{ABD} \cdot S_{ABC})) + (2.888887 \cdot (S_{ABD} \cdot S_{ABD})))}{(((0.666667 \cdot S_{ABC}) + (0.333333 \cdot S_{ABD})) + (0.444445 \cdot S_{ACD}))} \quad \text{by fractions on} \\ &\quad \text{two sides} \quad (65)
\end{aligned}$$

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16

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32

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[illegible]

[illegible]

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[illegible]

42

[illegible]

[illegible]

[illegible]

46

47

$$\begin{aligned}
& (((4.333335 \cdot (S_{ABC} + 0)) + (-4.333335 \cdot (8.666671 \cdot S_{ABC}))) + (4.333335 \cdot (S_{ABC} + 0))) && \text{by Lemma 2 (equal)} \quad (783) \\
& (((4.333335 \cdot (S_{ABC} + 0)) + (-4.333335 \cdot (8.666671 \cdot S_{ABC}))) + (4.333335 \cdot (S_{ABC} + 0))) && \text{by Lemma 1} \quad (784) \\
& (((4.333335 \cdot (S_{ABC} + 0)) + (-4.333335 \cdot (8.666671 \cdot S_{ABC}))) + (4.333335 \cdot (S_{ABC} + 0))) && \text{by Lemma 1} \quad (785) \\
& (((4.333335 \cdot S_{ABC}) + (-4.333335 \cdot (8.666671 \cdot S_{ABC}))) + (4.333335 \cdot (S_{ABC} + 0))) && \text{by addition with 0} \quad (786) \\
& (((4.333335 \cdot S_{ABC}) + (-4.333335 \cdot (8.666671 \cdot S_{ABC}))) + (4.333335 \cdot S_{ABC})) && \text{by addition with 0} \quad (787) \\
& (((4.333335 \cdot S_{ABC}) + (-4.333335 \cdot (8.666671 \cdot S_{ABC}))) + (4.333335 \cdot S_{ABC})) && \text{by multiplication of constants} \quad (788) \\
& (((8.666669 \cdot S_{ABC}) + (-4.333335 \cdot (8.666671 \cdot S_{ABC}))) + 0) && \text{by similar summands} \quad (789) \\
& ((8.666669 \cdot S_{ABC}) + (-4.333335 \cdot (8.666671 \cdot S_{ABC}))) && \text{by addition with 0} \quad (790) \\
& ((8.666669 \cdot S_{ABC}) + (-4.333335 \cdot (8.666671 \cdot S_{ABC}))) && \text{by similar summands} \quad (791) \\
& ((8.666669 \cdot S_{ABC}) + (-4.333335 \cdot (8.666671 \cdot S_{ABC}))) && \text{by multiplication by 0} \quad (792) \\
& ((8.666669 \cdot S_{ABC}) + (-4.333335 \cdot 0)) && \text{by addition with 0} \quad (793) \\
& ((8.666669 \cdot S_{ABC}) + 0) &= & (8.666671 \cdot S_{ABC}) && \text{by multiplication by 0} \quad (794) \\
& (8.666669 \cdot S_{ABC}) &= & (8.666671 \cdot S_{ABC}) && \text{by addition with 0} \quad (795) \\
& (0 \cdot S_{ABC}) &= & 0 && \text{by similar summands on two sides} \quad (796) \\
& 0 &= & 0 && \text{by multiplication by 0} \quad (797)
\end{aligned}$$

Q.E.D.

NDG conditions are:

$S_{AC}P_{cd}^0 \neq S_{P_{ad}^1}CP_{cd}^0$ i.e., lines AP_{ad}^1 and CP_{cd}^0 are not parallel (construction based assumption)

$S_{ABB_1} \neq S_{A_1BB_1}$ i.e., lines AA_1 and BB_1 are not parallel (construction based assumption)

$S_{ABC} \neq 0$ i.e., points A , B and C are not collinear (cancellation assumption)

Number of elimination proof steps: 62

Number of geometric proof steps: 152

Number of algebraic proof steps: 582

Total number of proof steps: 796
Time spent by the prover: 2.337 seconds