

$$(1) \quad \left( \left( \frac{\overrightarrow{AF}}{\overrightarrow{FB}} \cdot \frac{\overrightarrow{BD}}{\overrightarrow{DC}} \right) \cdot \frac{\overrightarrow{CE}}{\overrightarrow{EA}} \right) = 1 \quad , \quad \text{by the statement}$$

$$(2) \quad \left( \left( \left( -1 \cdot \frac{\overrightarrow{AF}}{\overrightarrow{BF}} \right) \cdot \frac{\overrightarrow{BD}}{\overrightarrow{DC}} \right) \cdot \frac{\overrightarrow{CE}}{\overrightarrow{EA}} \right) = 1 \quad , \quad \text{by geometric simplifications}$$

$$(3) \quad \left( -1 \cdot \left( \frac{\overrightarrow{AF}}{\overrightarrow{BF}} \cdot \left( \frac{\overrightarrow{BD}}{\overrightarrow{DC}} \cdot \frac{\overrightarrow{CE}}{\overrightarrow{EA}} \right) \right) \right) = 1 \quad , \quad \text{by algebraic simplifications}$$

$$(4) \quad \left( -1 \cdot \left( \frac{S_{APC}}{S_{BPC}} \cdot \left( \frac{\overrightarrow{BD}}{\overrightarrow{DC}} \cdot \frac{\overrightarrow{CE}}{\overrightarrow{EA}} \right) \right) \right) = 1 \quad , \quad \text{by Lemma 8 (point } F \text{ eliminated)}$$

1

$$(5) \quad \left( -1 \cdot \left( \frac{S_{APC}}{S_{BPC}} \cdot \left( \frac{\overrightarrow{BD}}{\overrightarrow{DC}} \cdot \left( -1 \cdot \frac{\overrightarrow{CE}}{\overrightarrow{AE}} \right) \right) \right) \right) = 1 \quad , \quad \text{by geometric simplifications}$$

$$(6) \quad \frac{\left( S_{APC} \cdot \left( \frac{\overrightarrow{BD}}{\overrightarrow{DC}} \cdot \frac{\overrightarrow{CE}}{\overrightarrow{AE}} \right) \right)}{S_{BPC}} = 1 \quad , \quad \text{by algebraic simplifications}$$

$$(7) \quad \frac{\left( S_{APC} \cdot \left( \frac{\overrightarrow{BD}}{\overrightarrow{DC}} \cdot \frac{S_{CPB}}{S_{APB}} \right) \right)}{S_{BPC}} = 1 \quad , \quad \text{by Lemma 8 (point } E \text{ eliminated)}$$

$$(8) \quad \frac{\left( S_{APC} \cdot \left( \left( -1 \cdot \frac{\overrightarrow{BD}}{\overrightarrow{CD}} \right) \cdot \frac{S_{CPB}}{S_{APB}} \right) \right)}{(-1 \cdot S_{CPB})} = 1 \quad , \quad \text{by geometric simplifications}$$

$$(9) \quad \frac{\left(S_{APC} \cdot \frac{\overrightarrow{BD}}{\overrightarrow{CD}}\right)}{S_{APB}} = 1 \quad , \quad \text{by algebraic simplifications}$$

$$(10) \quad \frac{\left(S_{APC} \cdot \frac{S_{BPA}}{S_{CPA}}\right)}{S_{APB}} = 1 \quad , \quad \text{by Lemma 8 (point } D \text{ eliminated)}$$

$$(11) \quad \frac{\left(S_{APC} \cdot \frac{S_{BPA}}{(-1 \cdot S_{APC})}\right)}{(-1 \cdot S_{BPA})} = 1 \quad , \quad \text{by geometric simplifications}$$

$$(12) \quad 1 = 1 \quad , \quad \text{by algebraic simplifications}$$

---

Q.E.D.

NDG conditions are:

$S_{BPA} \neq S_{CPA}$  i.e., lines  $BC$  and  $PA$  are not parallel (construction based assumption)

$S_{APB} \neq S_{CPB}$  i.e., lines  $AC$  and  $PB$  are not parallel (construction based assumption)

$S_{APC} \neq S_{BPC}$  i.e., lines  $AB$  and  $PC$  are not parallel (construction based assumption)

$P_{FBB} \neq 0$  i.e., points  $F$  and  $B$  are not identical (conjecture based assumption)

$P_{DCD} \neq 0$  i.e., points  $D$  and  $C$  are not identical (conjecture based assumption)

$P_{EAE} \neq 0$  i.e., points  $E$  and  $A$  are not identical (conjecture based assumption)

---

Number of elimination proof steps: 3

Number of geometric proof steps: 6

Number of algebraic proof steps: 23

Total number of proof steps: 32

Time spent by the prover: 0.000 seconds