Let F^0_{fg} be the foot of the normal from the point F on the line de.

GCLC Prover Output for conjecture “proof93”

Area method used

June 1, 2016

$$\frac{\overrightarrow{DG}}{\overrightarrow{GE}} = 1, \quad \text{by the statement} \tag{1}$$

$$\left(-1 \cdot \frac{\overrightarrow{DG}}{\overrightarrow{EG}}\right) = 1, \quad \text{by geometric simplifications} \tag{2}$$

$$\left(-1 \cdot \frac{S_{DFF^0_{fg}}}{S_{EFF^0_{fg}}}\right) = 1, \quad \text{by Lemma 8 (point } G \text{ eliminated)} \tag{3}$$

$$\frac{\left(-1 \cdot S_{DFF^0_{fg}}\right)}{S_{EFF^0_{fg}}} = 1, \quad \text{by algebraic simplifications} \tag{4}$$

$$\frac{\left(-1 \cdot \frac{((P_{FDE} \cdot S_{DFE}) + (P_{FED} \cdot S_{DFD}))}{P_{DED}}\right)}{S_{EFF^0_{fg}}} = 1, \quad \text{by Lemma 31 (point } F^0_{fg} \text{ eliminated)} \tag{5}$$

$$\frac{\left(-1 \cdot \frac{((P_{FDE} \cdot S_{DFE}) + (P_{FED} \cdot 0))}{P_{DED}}\right)}{S_{EFF^0_{fg}}} = 1, \quad \text{by geometric simplifications} \tag{6}$$

$$\frac{(-1 \cdot (P_{FDE} \cdot S_{DFE}))}{\left(P_{DED} \cdot S_{EFF^0_{fg}}\right)} = 1, \quad \text{by algebraic simplifications} \tag{7}$$

$$\frac{(-1 \cdot (P_{FDE} \cdot S_{DFE}))}{\left(P_{DED} \cdot \frac{((P_{FDE} \cdot S_{EFE}) + (P_{FED} \cdot S_{EFD}))}{P_{DED}}\right)} = 1, \quad \text{by Lemma 31 (point } F^0_{fg} \text{ eliminated)} \tag{8}$$

$$\frac{(-1 \cdot (P_{FDE} \cdot S_{DFE}))}{\left(P_{DED} \cdot \frac{((P_{FDE} \cdot 0)+(P_{FED} \cdot (-1 \cdot S_{DFE})))}{P_{DED}}\right)} = 1, \quad \text{by geometric simplifications} \tag{9}$$

$$\frac{P_{FDE}}{P_{FED}} = 1, \quad \text{by algebraic simplifications} \tag{10}$$

$$\frac{\frac{((P_{BAC} \cdot P_{FDC})+(P_{BCA} \cdot P_{FDA}))}{P_{ACA}}}{P_{FED}} = 1, \quad \text{by Lemma 31 (point } E \text{ eliminated)} \tag{11}$$

$$\frac{((P_{BAC} \cdot P_{FDC}) + (P_{BCA} \cdot P_{FDA}))}{(P_{ACA} \cdot P_{FED})} = 1, \quad \text{by algebraic simplifications} \tag{12}$$

$$\frac{((P_{BAC} \cdot P_{FDC}) + (P_{BCA} \cdot P_{FDA}))}{\left(P_{ACA} \cdot \left(\left(\left(\frac{\overrightarrow{AE}}{\overrightarrow{AC}} \cdot P_{FCD}\right) + \left(\frac{\overrightarrow{EC}}{\overrightarrow{AC}} \cdot P_{FAD}\right)\right) + \left(-1 \cdot \left(\left(\frac{\overrightarrow{AE}}{\overrightarrow{AC}} \cdot \frac{\overrightarrow{EC}}{\overrightarrow{AC}}\right) \cdot P_{ACA}\right)\right)\right)\right)} = 1, \quad \text{by Lemma 32 (point } E \text{ eliminated)} \tag{13}$$

$$\frac{((P_{BAC} \cdot P_{FDC}) + (P_{BCA} \cdot P_{FDA}))}{\left(P_{ACA} \cdot \left(\left(\left(\frac{\overrightarrow{AE}}{\overrightarrow{AC}} \cdot P_{FCD}\right) + \left(\left(-1 \cdot \frac{\overrightarrow{CE}}{\overrightarrow{AC}}\right) \cdot P_{FAD}\right)\right) + \left(-1 \cdot \left(\left(\frac{\overrightarrow{AE}}{\overrightarrow{AC}} \cdot \left(-1 \cdot \frac{\overrightarrow{CE}}{\overrightarrow{AC}}\right)\right) \cdot P_{ACA}\right)\right)\right)\right)}$$
$$= 1, \quad \text{by geometric simplifications} \tag{14}$$

$$\frac{((P_{BAC} \cdot P_{FDC}) + (P_{BCA} \cdot P_{FDA}))}{\left(\left(\left(P_{ACA} \cdot \left(\frac{\overrightarrow{AE}}{\overrightarrow{AC}} \cdot P_{FCD}\right)\right) + \left(-1 \cdot \left(P_{ACA} \cdot \left(\frac{\overrightarrow{CE}}{\overrightarrow{AC}} \cdot P_{FAD}\right)\right)\right)\right) + \left(P_{ACA} \cdot \left(\frac{\overrightarrow{AE}}{\overrightarrow{AC}} \cdot \left(\frac{\overrightarrow{CE}}{\overrightarrow{AC}} \cdot P_{ACA}\right)\right)\right)\right)}$$
$$= 1, \quad \text{by algebraic simplifications} \tag{15}$$

$$\frac{((P_{BAC} \cdot P_{FDC}) + (P_{BCA} \cdot P_{FDA}))}{\left(\left(\left(P_{ACA} \cdot \left(\frac{P_{BAAC}}{P_{ACA}} \cdot P_{FCD}\right)\right) + \left(-1 \cdot \left(P_{ACA} \cdot \left(\frac{\overrightarrow{CE}}{\overrightarrow{AC}} \cdot P_{FAD}\right)\right)\right)\right) + \left(P_{ACA} \cdot \left(\frac{\overrightarrow{AE}}{\overrightarrow{AC}} \cdot \left(\frac{\overrightarrow{CE}}{\overrightarrow{AC}} \cdot P_{ACA}\right)\right)\right)\right)} = 1, \quad \begin{array}{l}\text{by Lemma 38 , first case — points } A\text{, } A\text{, and}\\ C \text{ are collinear (point } E \text{ eliminated)}\end{array} \tag{16}$$

$$\frac{((P_{BAC} \cdot P_{FDC}) + (P_{BCA} \cdot P_{FDA}))}{\left(\left((P_{BAAC} \cdot P_{FCD}) + \left(-1 \cdot \left(P_{ACA} \cdot \left(\frac{\overrightarrow{CE}}{\overrightarrow{AC}} \cdot P_{FAD}\right)\right)\right)\right) + \left(P_{ACA} \cdot \left(\frac{\overrightarrow{AE}}{\overrightarrow{AC}} \cdot \left(\frac{\overrightarrow{CE}}{\overrightarrow{AC}} \cdot P_{ACA}\right)\right)\right)\right)} = 1, \quad \text{by algebraic simplifications} \tag{17}$$

$$\frac{((P_{BAC} \cdot P_{FDC}) + (P_{BCA} \cdot P_{FDA}))}{\left(\left((P_{BAAC} \cdot P_{FCD}) + \left(-1 \cdot \left(P_{ACA} \cdot \left(\frac{P_{BACC}}{P_{ACA}} \cdot P_{FAD}\right)\right)\right)\right) + \left(P_{ACA} \cdot \left(\frac{\overrightarrow{AE}}{\overrightarrow{AC}} \cdot \left(\frac{\overrightarrow{CE}}{\overrightarrow{AC}} \cdot P_{ACA}\right)\right)\right)\right)} = 1, \quad \begin{array}{l}\text{by Lemma 38 , first case — points } C\text{, } A\text{, and}\\ C \text{ are collinear (point } E \text{ eliminated)}\end{array} \tag{18}$$

$$\frac{((P_{BAC} \cdot P_{FDC}) + (P_{BCA} \cdot P_{FDA}))}{\left(((P_{BAAC} \cdot P_{FCD}) + (-1 \cdot (P_{BACC} \cdot P_{FAD}))) + \left(P_{ACA} \cdot \left(\frac{\overrightarrow{AE}}{\overrightarrow{AC}} \cdot \left(\frac{\overrightarrow{CE}}{\overrightarrow{AC}} \cdot P_{ACA}\right)\right)\right)\right)} = 1, \quad \text{by algebraic simplifications} \tag{19}$$

$$\frac{((P_{BAC} \cdot P_{FDC}) + (P_{BCA} \cdot P_{FDA}))}{\left(((P_{BAAC} \cdot P_{FCD}) + (-1 \cdot (P_{BACC} \cdot P_{FAD}))) + \left(P_{ACA} \cdot \left(\frac{P_{BAAC}}{P_{ACA}} \cdot \left(\frac{\overrightarrow{CE}}{\overrightarrow{AC}} \cdot P_{ACA}\right)\right)\right)\right)} = 1, \quad \begin{array}{l}\text{by Lemma 38 , first case — points } A\text{, } A\text{, and}\\ C \text{ are collinear (point } E \text{ eliminated)}\end{array} \tag{20}$$

$$\frac{((P_{BAC} \cdot P_{FDC}) + (P_{BCA} \cdot P_{FDA}))}{\left(((P_{BAAC} \cdot P_{FCD}) + (-1 \cdot (P_{BACC} \cdot P_{FAD}))) + \left(P_{ACA} \cdot \left(P_{BAAC} \cdot \frac{\overrightarrow{CE}}{\overrightarrow{AC}}\right)\right)\right)} = 1, \quad \text{by algebraic simplifications} \tag{21}$$

$$\frac{((P_{BAC}\cdot P_{FDC})+(P_{BCA}\cdot P_{FDA}))}{\left(((P_{BAAC}\cdot P_{FCD})+(-1\cdot(P_{BACC}\cdot P_{FAD})))+\left(P_{ACA}\cdot\left(P_{BAAC}\cdot\frac{P_{BACC}}{P_{ACA}}\right)\right)\right)}=1, \tag{22}$$
by Lemma 38 , first case — points C, A, and C are collinear (point E eliminated)

$$\frac{((P_{BAC}\cdot P_{FDC})+(P_{BCA}\cdot P_{FDA}))}{(((P_{BAAC}\cdot P_{FCD})+(-1\cdot(P_{BACC}\cdot P_{FAD})))+(P_{BAAC}\cdot P_{BACC}))}=1, \tag{23}$$
by algebraic simplifications

$$\frac{\left(\left(P_{BAC}\cdot\left(\left(\left(\frac{\overrightarrow{AD}}{\overrightarrow{AB}}\cdot P_{FBC}\right)+\left(\frac{\overrightarrow{DB}}{\overrightarrow{AB}}\cdot P_{FAC}\right)\right)+\left(-1\cdot\left(\left(\frac{\overrightarrow{AD}}{\overrightarrow{AB}}\cdot\frac{\overrightarrow{DB}}{\overrightarrow{AB}}\right)\cdot P_{ABA}\right)\right)\right)\right)+(P_{BCA}\cdot P_{FDA})\right)}{(((P_{BAAC}\cdot P_{FCD})+(-1\cdot(P_{BACC}\cdot P_{FAD})))+(P_{BAAC}\cdot P_{BACC}))}=1, \tag{24}$$
by Lemma 32 (point D eliminated)

$$\frac{\left(\left(P_{BAC}\cdot\left(\left(\left(\frac{\overrightarrow{AD}}{\overrightarrow{AB}}\cdot P_{FBC}\right)+\left(\left(-1\cdot\frac{\overrightarrow{BD}}{\overrightarrow{AB}}\right)\cdot P_{FAC}\right)\right)+\left(-1\cdot\left(\left(\frac{\overrightarrow{AD}}{\overrightarrow{AB}}\cdot\left(-1\cdot\frac{\overrightarrow{BD}}{\overrightarrow{AB}}\right)\right)\cdot P_{ABA}\right)\right)\right)\right)+(P_{BCA}\cdot P_{FDA})\right)}{(((P_{BAAC}\cdot P_{FCD})+(-1\cdot(P_{BACC}\cdot P_{FAD})))+(P_{BAAC}\cdot P_{BACC}))}=1, \tag{25}$$
by geometric simplifications

$$\frac{\left(\left(\left(\left(P_{BAC}\cdot\left(\frac{\overrightarrow{AD}}{\overrightarrow{AB}}\cdot P_{FBC}\right)\right)+\left(-1\cdot\left(P_{BAC}\cdot\left(\frac{\overrightarrow{BD}}{\overrightarrow{AB}}\cdot P_{FAC}\right)\right)\right)\right)+\left(P_{BAC}\cdot\left(\frac{\overrightarrow{AD}}{\overrightarrow{AB}}\cdot\left(\frac{\overrightarrow{BD}}{\overrightarrow{AB}}\cdot P_{ABA}\right)\right)\right)\right)+(P_{BCA}\cdot P_{FDA})\right)}{(((P_{BAAC}\cdot P_{FCD})+(-1\cdot(P_{BACC}\cdot P_{FAD})))+(P_{BAAC}\cdot P_{BACC}))}=1, \tag{26}$$
by algebraic simplifications

$$\frac{\left(\left(\left(\left(P_{BAC}\cdot\left(\frac{P_{CAAB}}{P_{ABA}}\cdot P_{FBC}\right)\right)+\left(-1\cdot\left(P_{BAC}\cdot\left(\frac{\overrightarrow{BD}}{\overrightarrow{AB}}\cdot P_{FAC}\right)\right)\right)\right)+\left(P_{BAC}\cdot\left(\frac{\overrightarrow{AD}}{\overrightarrow{AB}}\cdot\left(\frac{\overrightarrow{BD}}{\overrightarrow{AB}}\cdot P_{ABA}\right)\right)\right)\right)+(P_{BCA}\cdot P_{FDA})\right)}{(((P_{BAAC}\cdot P_{FCD})+(-1\cdot(P_{BACC}\cdot P_{FAD})))+(P_{BAAC}\cdot P_{BACC}))}=1, \tag{27}$$
by Lemma 38 , first case — points A, A, and B are collinear (point D eliminated)

$$\frac{\left(\left(\left((P_{BAC}\cdot(P_{CAAB}\cdot P_{FBC}))+\left(-1\cdot\left(P_{BAC}\cdot\left(\frac{\overrightarrow{BD}}{\overrightarrow{AB}}\cdot(P_{FAC}\cdot P_{ABA})\right)\right)\right)\right)+\left(P_{BAC}\cdot\left(\frac{\overrightarrow{AD}}{\overrightarrow{AB}}\cdot\left(\frac{\overrightarrow{BD}}{\overrightarrow{AB}}\cdot(P_{ABA}\cdot P_{ABA})\right)\right)\right)\right)+(P_{BCA}\cdot(P_{FDA}\cdot P_{ABA}))\right)}{(((P_{ABA}\cdot(P_{BAAC}\cdot P_{FCD}))+(-1\cdot(P_{ABA}\cdot(P_{BACC}\cdot P_{FAD}))))+(P_{ABA}\cdot(P_{BAAC}\cdot P_{BACC})))} \tag{28}$$

$= 1,$ by algebraic simplifications

$$\frac{\left(\left(\left((P_{BAC}\cdot(P_{CAAB}\cdot P_{FBC}))+\left(-1\cdot\left(P_{BAC}\cdot\left(\frac{P_{CABB}}{P_{ABA}}\cdot(P_{FAC}\cdot P_{ABA})\right)\right)\right)\right)+\left(P_{BAC}\cdot\left(\frac{\overrightarrow{AD}}{\overrightarrow{AB}}\cdot\left(\frac{\overrightarrow{BD}}{\overrightarrow{AB}}\cdot(P_{ABA}\cdot P_{ABA})\right)\right)\right)\right)+(P_{BCA}\cdot(P_{FDA}\cdot P_{ABA}))\right)}{(((P_{ABA}\cdot(P_{BAAC}\cdot P_{FCD}))+(-1\cdot(P_{ABA}\cdot(P_{BACC}\cdot P_{FAD}))))+(P_{ABA}\cdot(P_{BAAC}\cdot P_{BACC})))} \tag{29}$$

$= 1,$ by Lemma 38 , first case — points B, A, and B are collinear (point D eliminated)

$$\frac{\left(\left(((P_{BAC}\cdot(P_{CAAB}\cdot P_{FBC}))+(-1\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{FAC}))))+\left(P_{BAC}\cdot\left(\frac{\overrightarrow{AD}}{\overrightarrow{AB}}\cdot\left(\frac{\overrightarrow{BD}}{\overrightarrow{AB}}\cdot(P_{ABA}\cdot P_{ABA})\right)\right)\right)\right)+(P_{BCA}\cdot(P_{FDA}\cdot P_{ABA}))\right)}{(((P_{ABA}\cdot(P_{BAAC}\cdot P_{FCD}))+(-1\cdot(P_{ABA}\cdot(P_{BACC}\cdot P_{FAD}))))+(P_{ABA}\cdot(P_{BAAC}\cdot P_{BACC})))} \tag{30}$$

$= 1,$ by algebraic simplifications

$$\frac{\left(\left(((P_{BAC}\cdot(P_{CAAB}\cdot P_{FBC}))+(-1\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{FAC}))))+\left(P_{BAC}\cdot\left(\frac{P_{CAAB}}{P_{ABA}}\cdot\left(\frac{\overrightarrow{BD}}{\overrightarrow{AB}}\cdot(P_{ABA}\cdot P_{ABA})\right)\right)\right)\right)+(P_{BCA}\cdot(P_{FDA}\cdot P_{ABA}))\right)}{(((P_{ABA}\cdot(P_{BAAC}\cdot P_{FCD}))+(-1\cdot(P_{ABA}\cdot(P_{BACC}\cdot P_{FAD}))))+(P_{ABA}\cdot(P_{BAAC}\cdot P_{BACC})))} \tag{31}$$

$= 1,$ by Lemma 38 , first case — points A, A, and B are collinear (point D eliminated)

$$\frac{\left(\left(((P_{BAC}\cdot(P_{CAAB}\cdot P_{FBC}))+(-1\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{FAC}))))+\left(P_{BAC}\cdot\left(P_{CAAB}\cdot\left(\frac{\overrightarrow{BD}}{\overrightarrow{AB}}\cdot P_{ABA}\right)\right)\right)\right)+(P_{BCA}\cdot(P_{FDA}\cdot P_{ABA}))\right)}{(((P_{ABA}\cdot(P_{BAAC}\cdot P_{FCD}))+(-1\cdot(P_{ABA}\cdot(P_{BACC}\cdot P_{FAD}))))+(P_{ABA}\cdot(P_{BAAC}\cdot P_{BACC})))} \tag{32}$$

$= 1,$ by algebraic simplifications

$$\frac{\Big(\Big(((P_{BAC}\cdot(P_{CAAB}\cdot P_{FBC}))+(-1\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{FAC}))))+\Big(P_{BAC}\cdot\Big(P_{CAAB}\cdot\Big(\frac{P_{CABB}}{P_{ABA}}\cdot P_{ABA}\Big)\Big)\Big)\Big)+(P_{BCA}\cdot(P_{FDA}\cdot P_{ABA}))\Big)}{(((P_{ABA}\cdot(P_{BAAC}\cdot P_{FCD}))+(-1\cdot(P_{ABA}\cdot(P_{BACC}\cdot P_{FAD}))))+(P_{ABA}\cdot(P_{BAAC}\cdot P_{BACC})))} \tag{33}$$

$= 1$, by Lemma 38 , first case — points B, A, and B are collinear (point D eliminated)

$$\frac{((((P_{BAC}\cdot(P_{CAAB}\cdot P_{FBC}))+(-1\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{FAC}))))+(P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB})))+(P_{BCA}\cdot(P_{FDA}\cdot P_{ABA})))}{(((P_{ABA}\cdot(P_{BAAC}\cdot P_{FCD}))+(-1\cdot(P_{ABA}\cdot(P_{BACC}\cdot P_{FAD}))))+(P_{ABA}\cdot(P_{BAAC}\cdot P_{BACC})))} \tag{34}$$

$= 1$, by algebraic simplifications

$$\frac{\Big((((P_{BAC}\cdot(P_{CAAB}\cdot P_{FBC}))+(-1\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{FAC}))))+(P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB})))+\Big(P_{BCA}\cdot\Big(\Big(\Big(\Big(\frac{\overrightarrow{AD}}{\overrightarrow{AB}}\cdot P_{FBA}\Big)+\Big(\frac{\overrightarrow{DB}}{\overrightarrow{AB}}\cdot P_{FAA}\Big)\Big)+\Big(-1\cdot\Big(\Big(\frac{\overrightarrow{AD}}{\overrightarrow{AB}}\cdot\frac{\overrightarrow{DB}}{\overrightarrow{AB}}\Big)}{(((P_{ABA}\cdot(P_{BAAC}\cdot P_{FCD}))+(-1\cdot(P_{ABA}\cdot(P_{BACC}\cdot P_{FAD}))))+(P_{ABA}\cdot(P_{BAAC}\cdot P_{BACC})))} \tag{35}$$

$= 1$, by Lemma 32 (point D eliminated)

$$\frac{\Big((((P_{BAC}\cdot(P_{CAAB}\cdot P_{FBC}))+(-1\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{FAC}))))+(P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB})))+\Big(P_{BCA}\cdot\Big(\Big(\Big(\Big(\frac{\overrightarrow{AD}}{\overrightarrow{AB}}\cdot P_{FBA}\Big)+\Big(\Big(-1\cdot\frac{\overrightarrow{BD}}{\overrightarrow{AB}}\Big)\cdot 0\Big)\Big)+\Big(-1\cdot\Big(\Big(\frac{\overrightarrow{AD}}{\overrightarrow{AB}}\cdot\Big(-}{(((P_{ABA}\cdot(P_{BAAC}\cdot P_{FCD}))+(-1\cdot(P_{ABA}\cdot(P_{BACC}\cdot P_{FAD}))))+(P_{ABA}\cdot(P_{BAAC}\cdot P_{BACC})))} \tag{36}$$

$= 1$, by geometric simplifications

$$\frac{\Big((((P_{BAC}\cdot(P_{CAAB}\cdot P_{FBC}))+(-1\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{FAC}))))+(P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB})))+\Big(\Big(P_{BCA}\cdot\Big(P_{ABA}\cdot\Big(\frac{\overrightarrow{AD}}{\overrightarrow{AB}}\cdot P_{FBA}\Big)\Big)\Big)+\Big(P_{BCA}\cdot\Big(P_{ABA}\cdot\Big(\frac{\overrightarrow{AD}}{\overrightarrow{AB}}\cdot\Big(\frac{\overrightarrow{BD}}{\overrightarrow{AB}}}{(((P_{ABA}\cdot(P_{BAAC}\cdot P_{FCD}))+(-1\cdot(P_{ABA}\cdot(P_{BACC}\cdot P_{FAD}))))+(P_{ABA}\cdot(P_{BAAC}\cdot P_{BACC})))} \tag{37}$$

$= 1$, by algebraic simplifications

$$\frac{\Big((((P_{BAC}\cdot(P_{CAAB}\cdot P_{FBC}))+(-1\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{FAC}))))+(P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB})))+\Big(\Big(P_{BCA}\cdot\Big(P_{ABA}\cdot\Big(\frac{P_{CAAB}}{P_{ABA}}\cdot P_{FBA}\Big)\Big)\Big)+\Big(P_{BCA}\cdot\Big(P_{ABA}\cdot\Big(\frac{\overrightarrow{AD}}{\overrightarrow{AB}}\cdot\Big(\frac{\overrightarrow{BD}}{\overrightarrow{AB}}}{(((P_{ABA}\cdot(P_{BAAC}\cdot P_{FCD}))+(-1\cdot(P_{ABA}\cdot(P_{BACC}\cdot P_{FAD}))))+(P_{ABA}\cdot(P_{BAAC}\cdot P_{BACC})))} \tag{38}$$

$= 1$, by Lemma 38 , first case — points A, A, and B are collinear (point D eliminated)

$$\frac{\Bigg((((P_{BAC}\cdot(P_{CAAB}\cdot P_{FBC}))+(-1\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{FAC}))))+(P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB})))+\Bigg((P_{BCA}\cdot(P_{CAAB}\cdot P_{FBA}))+\Bigg(P_{BCA}\cdot\Bigg(P_{ABA}\cdot\Bigg(\frac{\overrightarrow{AD}}{\overrightarrow{AB}}\cdot\Bigg(\frac{\overrightarrow{BD}}{\overrightarrow{AB}}\cdot P_{ABA}\Bigg)\Bigg)\Bigg)}{(((P_{ABA}\cdot(P_{BAAC}\cdot P_{FCD}))+(-1\cdot(P_{ABA}\cdot(P_{BACC}\cdot P_{FAD}))))+(P_{ABA}\cdot(P_{BAAC}\cdot P_{BACC})))} \tag{39}$$

$= 1,$ by algebraic simplifications

$$\frac{\Bigg((((P_{BAC}\cdot(P_{CAAB}\cdot P_{FBC}))+(-1\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{FAC}))))+(P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB})))+\Bigg((P_{BCA}\cdot(P_{CAAB}\cdot P_{FBA}))+\Bigg(P_{BCA}\cdot\Bigg(P_{ABA}\cdot\Bigg(\frac{P_{CAAB}}{P_{ABA}}\cdot\Bigg(\frac{\overrightarrow{BD}}{\overrightarrow{AB}}\cdot P_{ABA}\Bigg)}{(((P_{ABA}\cdot(P_{BAAC}\cdot P_{FCD}))+(-1\cdot(P_{ABA}\cdot(P_{BACC}\cdot P_{FAD}))))+(P_{ABA}\cdot(P_{BAAC}\cdot P_{BACC})))} \tag{40}$$

$= 1,$ by Lemma 38 , first case — points A, A, and B are collinear (point D eliminated)

$$\frac{\Bigg((((P_{BAC}\cdot(P_{CAAB}\cdot P_{FBC}))+(-1\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{FAC}))))+(P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB})))+\Bigg((P_{BCA}\cdot(P_{CAAB}\cdot P_{FBA}))+\Bigg(P_{BCA}\cdot\Bigg(P_{ABA}\cdot\Bigg(P_{CAAB}\cdot\frac{\overrightarrow{BD}}{\overrightarrow{AB}}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)}{(((P_{ABA}\cdot(P_{BAAC}\cdot P_{FCD}))+(-1\cdot(P_{ABA}\cdot(P_{BACC}\cdot P_{FAD}))))+(P_{ABA}\cdot(P_{BAAC}\cdot P_{BACC})))} \tag{41}$$

$= 1,$ by algebraic simplifications

$$\frac{\Big((((P_{BAC}\cdot(P_{CAAB}\cdot P_{FBC}))+(-1\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{FAC}))))+(P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB})))+\Big((P_{BCA}\cdot(P_{CAAB}\cdot P_{FBA}))+\Big(P_{BCA}\cdot\Big(P_{ABA}\cdot\Big(P_{CAAB}\cdot\frac{P_{CABB}}{P_{ABA}}\Big)\Big)\Big)\Big)\Big)}{(((P_{ABA}\cdot(P_{BAAC}\cdot P_{FCD}))+(-1\cdot(P_{ABA}\cdot(P_{BACC}\cdot P_{FAD}))))+(P_{ABA}\cdot(P_{BAAC}\cdot P_{BACC})))} \tag{42}$$

$= 1,$ by Lemma 38 , first case — points B, A, and B are collinear (point D eliminated)

$$\frac{((((P_{BAC}\cdot(P_{CAAB}\cdot P_{FBC}))+(-1\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{FAC}))))+(P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB})))+((P_{BCA}\cdot(P_{CAAB}\cdot P_{FBA}))+(P_{BCA}\cdot(P_{CAAB}\cdot P_{CABB}))))}{(((P_{ABA}\cdot(P_{BAAC}\cdot P_{FCD}))+(-1\cdot(P_{ABA}\cdot(P_{BACC}\cdot P_{FAD}))))+(P_{ABA}\cdot(P_{BAAC}\cdot P_{BACC})))} \tag{43}$$

$= 1,$ by algebraic simplifications

$$\frac{((((P_{BAC}\cdot(P_{CAAB}\cdot P_{FBC}))+(-1\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{FAC}))))+(P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB})))+((P_{BCA}\cdot(P_{CAAB}\cdot P_{FBA}))+(P_{BCA}\cdot(P_{CAAB}\cdot P_{CABB}))))}{\Big(\Big(\Big(P_{ABA}\cdot\Big(P_{BAAC}\cdot\frac{((P_{CAB}\cdot P_{FCB})+(P_{CBA}\cdot P_{FCA}))}{P_{ABA}}\Big)\Big)+(-1\cdot(P_{ABA}\cdot(P_{BACC}\cdot P_{FAD})))\Big)+(P_{ABA}\cdot(P_{BAAC}\cdot P_{BACC}))\Big)} \tag{44}$$

$= 1,$ by Lemma 31 (point D eliminated)

(45) $$\frac{((((P_{BAC} \cdot (P_{CAAB} \cdot P_{FBC})) + (-1 \cdot (P_{BAC} \cdot (P_{CABB} \cdot P_{FAC})))) + (P_{BAC} \cdot (P_{CAAB} \cdot P_{CABB}))) + ((P_{BCA} \cdot (P_{CAAB} \cdot P_{FBA})) + (P_{BCA} \cdot (P_{CAAB} \cdot P_{CABB}))))}{\left(\left(\left(P_{BAB} \cdot \left(P_{BAAC} \cdot \frac{((P_{BAC} \cdot P_{FCB}) + (P_{CBA} \cdot P_{FCA}))}{P_{BAB}}\right)\right) + (-1 \cdot (P_{BAB} \cdot (P_{BACC} \cdot P_{FAD})))\right) + (P_{BAB} \cdot (P_{BAAC} \cdot P_{BACC}))\right)}$$
$= 1$, by geometric simplifications

(46) $$\frac{((((P_{BAC} \cdot (P_{CAAB} \cdot P_{FBC})) + (-1 \cdot (P_{BAC} \cdot (P_{CABB} \cdot P_{FAC})))) + (P_{BAC} \cdot (P_{CAAB} \cdot P_{CABB}))) + ((P_{BCA} \cdot (P_{CAAB} \cdot P_{FBA})) + (P_{BCA} \cdot (P_{CAAB} \cdot P_{CABB}))))}{((((P_{BAAC} \cdot (P_{BAC} \cdot P_{FCB})) + (P_{BAAC} \cdot (P_{CBA} \cdot P_{FCA}))) + (-1 \cdot (P_{BAB} \cdot (P_{BACC} \cdot P_{FAD})))) + (P_{BAB} \cdot (P_{BAAC} \cdot P_{BACC})))}$$
$= 1$, by algebraic simplifications

(47) $$\frac{((((P_{BAC} \cdot (P_{CAAB} \cdot P_{FBC})) + (-1 \cdot (P_{BAC} \cdot (P_{CABB} \cdot P_{FAC})))) + (P_{BAC} \cdot (P_{CAAB} \cdot P_{CABB}))) + ((P_{BCA} \cdot (P_{CAAB} \cdot P_{FBA})) + (P_{BCA} \cdot (P_{CAAB} \cdot P_{CABB}))))}{\left(\left(((P_{BAAC} \cdot (P_{BAC} \cdot P_{FCB})) + (P_{BAAC} \cdot (P_{CBA} \cdot P_{FCA}))) + \left(-1 \cdot \left(P_{BAB} \cdot \left(P_{BACC} \cdot \frac{((P_{CAB} \cdot P_{FAB}) + (P_{CBA} \cdot P_{FAA}))}{P_{ABA}}\right)\right)\right)\right) + (P_{BAB} \cdot (P_{BAAC} \cdot P_{BACC}))\right)}$$
$= 1$, by Lemma 31 (point D eliminated)

(48) $$\frac{((((P_{BAC} \cdot (P_{CAAB} \cdot P_{FBC})) + (-1 \cdot (P_{BAC} \cdot (P_{CABB} \cdot P_{FAC})))) + (P_{BAC} \cdot (P_{CAAB} \cdot P_{CABB}))) + ((P_{BCA} \cdot (P_{CAAB} \cdot P_{FBA})) + (P_{BCA} \cdot (P_{CAAB} \cdot P_{CABB}))))}{\left(\left(((P_{BAAC} \cdot (P_{BAC} \cdot P_{FCB})) + (P_{BAAC} \cdot (P_{CBA} \cdot P_{FCA}))) + \left(-1 \cdot \left(P_{BAB} \cdot \left(P_{BACC} \cdot \frac{((P_{BAC} \cdot P_{FAB}) + (P_{CBA} \cdot 0))}{P_{BAB}}\right)\right)\right)\right) + (P_{BAB} \cdot (P_{BAAC} \cdot P_{BACC}))\right)}$$
$= 1$, by geometric simplifications

(49) $$\frac{((((P_{BAC} \cdot (P_{CAAB} \cdot P_{FBC})) + (-1 \cdot (P_{BAC} \cdot (P_{CABB} \cdot P_{FAC})))) + (P_{BAC} \cdot (P_{CAAB} \cdot P_{CABB}))) + ((P_{BCA} \cdot (P_{CAAB} \cdot P_{FBA})) + (P_{BCA} \cdot (P_{CAAB} \cdot P_{CABB}))))}{((((P_{BAAC} \cdot (P_{BAC} \cdot P_{FCB})) + (P_{BAAC} \cdot (P_{CBA} \cdot P_{FCA}))) + (-1 \cdot (P_{BACC} \cdot (P_{BAC} \cdot P_{FAB})))) + (P_{BAB} \cdot (P_{BAAC} \cdot P_{BACC})))}$$
$= 1$, by algebraic simplifications

(50) $$\frac{((((P_{BAC} \cdot (P_{CAAB} \cdot P_{CBF})) + (-1 \cdot (P_{BAC} \cdot (P_{CABB} \cdot P_{CAF})))) + (P_{BAC} \cdot (P_{CAAB} \cdot P_{CABB}))) + ((P_{BCA} \cdot (P_{CAAB} \cdot P_{ABF})) + (P_{BCA} \cdot (P_{CAAB} \cdot P_{CABB}))))}{((((P_{BAAC} \cdot (P_{BAC} \cdot P_{BCF})) + (P_{BAAC} \cdot (P_{CBA} \cdot P_{ACF}))) + (-1 \cdot (P_{BACC} \cdot (P_{BAC} \cdot P_{BAF})))) + (P_{BAB} \cdot (P_{BAAC} \cdot P_{BACC})))}$$
$= 1$, by geometric simplifications

(51) $$\frac{((((P_{BAC} \cdot (P_{CAAB} \cdot (P_{CBB} + (\frac{1}{2} \cdot (P_{CBC} + (-1 \cdot P_{CBB})))))) + (-1 \cdot (P_{BAC} \cdot (P_{CABB} \cdot P_{CAF})))) + (P_{BAC} \cdot (P_{CAAB} \cdot P_{CABB}))) + ((P_{BCA} \cdot (P_{CAAB} \cdot P_{ABF})) + (P_{BCA} \cdot (P}{((((P_{BAAC} \cdot (P_{BAC} \cdot P_{BCF})) + (P_{BAAC} \cdot (P_{CBA} \cdot P_{ACF}))) + (-1 \cdot (P_{BACC} \cdot (P_{BAC} \cdot P_{BAF})))) + (P_{BAB} \cdot (P_{BAAC} \cdot P_{BACC})))}$$
$= 1$, by Lemma 29 (point F eliminated)

$$\frac{((((P_{BAC} \cdot (P_{CAAB} \cdot (0 + (\frac{1}{2} \cdot (P_{CBC} + (-1 \cdot 0))))))) + (-1 \cdot (P_{BAC} \cdot (P_{CABB} \cdot P_{CAF})))) + (P_{BAC} \cdot (P_{CAAB} \cdot P_{CABB}))) + ((P_{BCA} \cdot (P_{CAAB} \cdot P_{ABF})) + (P_{BCA} \cdot (P_{CAAB} \cdot P_{C}\ldots}{((((P_{BAAC} \cdot (P_{BAC} \cdot P_{BCF})) + (P_{BAAC} \cdot (P_{CBA} \cdot P_{ACF}))) + (-1 \cdot (P_{BACC} \cdot (P_{BAC} \cdot P_{BAF})))) + (P_{BAB} \cdot (P_{BAAC} \cdot P_{BACC})))} \tag{52}$$
$= 1,$ by geometric simplifications

$$\frac{((((\frac{1}{2} \cdot (P_{BAC} \cdot (P_{CAAB} \cdot P_{CBC}))) + (-1 \cdot (P_{BAC} \cdot (P_{CABB} \cdot P_{CAF})))) + (P_{BAC} \cdot (P_{CAAB} \cdot P_{CABB}))) + ((P_{BCA} \cdot (P_{CAAB} \cdot P_{ABF})) + (P_{BCA} \cdot (P_{CAAB} \cdot P_{CABB}))))}{((((P_{BAAC} \cdot (P_{BAC} \cdot P_{BCF})) + (P_{BAAC} \cdot (P_{CBA} \cdot P_{ACF}))) + (-1 \cdot (P_{BACC} \cdot (P_{BAC} \cdot P_{BAF})))) + (P_{BAB} \cdot (P_{BAAC} \cdot P_{BACC})))} \tag{53}$$
$= 1,$ by algebraic simplifications

$$\frac{((((\frac{1}{2} \cdot (P_{BAC} \cdot (P_{CAAB} \cdot P_{CBC}))) + (-1 \cdot (P_{BAC} \cdot (P_{CABB} \cdot (P_{CAB} + (\frac{1}{2} \cdot (P_{CAC} + (-1 \cdot P_{CAB})))))))) + (P_{BAC} \cdot (P_{CAAB} \cdot P_{CABB}))) + ((P_{BCA} \cdot (P_{CAAB} \cdot P_{ABF})) + (P_{BC}\ldots}{((((P_{BAAC} \cdot (P_{BAC} \cdot P_{BCF})) + (P_{BAAC} \cdot (P_{CBA} \cdot P_{ACF}))) + (-1 \cdot (P_{BACC} \cdot (P_{BAC} \cdot P_{BAF})))) + (P_{BAB} \cdot (P_{BAAC} \cdot P_{BACC})))} \tag{54}$$
$= 1,$ by Lemma 29 (point F eliminated)

$$\frac{((((\frac{1}{2} \cdot (P_{BAC} \cdot (P_{CAAB} \cdot P_{CBC}))) + (-1 \cdot (P_{BAC} \cdot (P_{CABB} \cdot (P_{BAC} + (\frac{1}{2} \cdot (P_{CAC} + (-1 \cdot P_{BAC})))))))) + (P_{BAC} \cdot (P_{CAAB} \cdot P_{CABB}))) + ((P_{BCA} \cdot (P_{CAAB} \cdot P_{ABF})) + (P_{BC}\ldots}{((((P_{BAAC} \cdot (P_{BAC} \cdot P_{BCF})) + (P_{BAAC} \cdot (P_{CBA} \cdot P_{ACF}))) + (-1 \cdot (P_{BACC} \cdot (P_{BAC} \cdot P_{BAF})))) + (P_{BAB} \cdot (P_{BAAC} \cdot P_{BACC})))} \tag{55}$$
$= 1,$ by geometric simplifications

$$\frac{((((\frac{1}{2} \cdot (P_{BAC} \cdot (P_{CAAB} \cdot P_{CBC}))) + ((-\frac{1}{2} \cdot (P_{BAC} \cdot (P_{CABB} \cdot P_{BAC}))) + (-\frac{1}{2} \cdot (P_{BAC} \cdot (P_{CABB} \cdot P_{CAC}))))) + (P_{BAC} \cdot (P_{CAAB} \cdot P_{CABB}))) + ((P_{BCA} \cdot (P_{CAAB} \cdot P_{ABF})) + (\ldots}{((((P_{BAAC} \cdot (P_{BAC} \cdot P_{BCF})) + (P_{BAAC} \cdot (P_{CBA} \cdot P_{ACF}))) + (-1 \cdot (P_{BACC} \cdot (P_{BAC} \cdot P_{BAF})))) + (P_{BAB} \cdot (P_{BAAC} \cdot P_{BACC})))} \tag{56}$$
$= 1,$ by algebraic simplifications

$$\frac{((((\frac{1}{2} \cdot (P_{BAC} \cdot (P_{CAAB} \cdot P_{CBC}))) + ((-\frac{1}{2} \cdot (P_{BAC} \cdot (P_{CABB} \cdot P_{BAC}))) + (-\frac{1}{2} \cdot (P_{BAC} \cdot (P_{CABB} \cdot P_{CAC}))))) + (P_{BAC} \cdot (P_{CAAB} \cdot P_{CABB}))) + ((P_{BCA} \cdot (P_{CAAB} \cdot (P_{ABB} + (\ldots}{((((P_{BAAC} \cdot (P_{BAC} \cdot P_{BCF})) + (P_{BAAC} \cdot (P_{CBA} \cdot P_{ACF}))) + (-1 \cdot (P_{BACC} \cdot (P_{BAC} \cdot P_{BAF})))) + (P_{BAB} \cdot (P_{BAAC} \cdot P_{B}\ldots} \tag{57}$$
$= 1,$ by Lemma 29 (point F eliminated)

$$\frac{((((\frac{1}{2} \cdot (P_{BAC} \cdot (P_{CAAB} \cdot P_{CBC}))) + ((-\frac{1}{2} \cdot (P_{BAC} \cdot (P_{CABB} \cdot P_{BAC}))) + (-\frac{1}{2} \cdot (P_{BAC} \cdot (P_{CABB} \cdot P_{CAC}))))) + (P_{BAC} \cdot (P_{CAAB} \cdot P_{CABB}))) + ((P_{BCA} \cdot (P_{CAAB} \cdot (0 + (\frac{1}{2} \cdot (\ldots}{((((P_{BAAC} \cdot (P_{BAC} \cdot P_{BCF})) + (P_{BAAC} \cdot (P_{ABC} \cdot P_{ACF}))) + (-1 \cdot (P_{BACC} \cdot (P_{BAC} \cdot P_{BAF})))) + (P_{ABA} \cdot (P_{BAAC} \cdot P_{BACC})\ldots} \tag{58}$$
$= 1,$ by geometric simplifications

$$\frac{((((\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CAAB}\cdot P_{CBC}))) + ((-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{BAC}))) + (-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{CAC}))))) + (P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB}))) + ((\frac{1}{2}\cdot(P_{BCA}\cdot(P_{CAAB}\cdot P_{ABC})}{((((P_{BAAC}\cdot(P_{BAC}\cdot P_{BCF})) + (P_{BAAC}\cdot(P_{ABC}\cdot P_{ACF}))) + (-1\cdot(P_{BACC}\cdot(P_{BAC}\cdot P_{BAF})))) + (P_{ABA}\cdot(P_{BAAC}\cdot P_{BACC})))} \tag{59}$$

$= 1,\quad$ by algebraic simplifications

$$\frac{((((\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CAAB}\cdot P_{CBC}))) + ((-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{BAC}))) + (-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{CAC}))))) + (P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB}))) + ((\frac{1}{2}\cdot(P_{BCA}\cdot(P_{CAAB}\cdot P_{ABC})}{((((P_{BAAC}\cdot(P_{BAC}\cdot(P_{BCB}+(\frac{1}{2}\cdot(P_{BCC}+(-1\cdot P_{BCB}))))))) + (P_{BAAC}\cdot(P_{ABC}\cdot P_{ACF}))) + (-1\cdot(P_{BACC}\cdot(P_{BAC}\cdot P_{BAF})))) + (P_{ABA}\cdot(P_{BAAC}\cdot} \tag{60}$$

$= 1,\quad$ by Lemma 29 (point F eliminated)

$$\frac{((((\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CAAB}\cdot P_{CBC}))) + ((-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{BAC}))) + (-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{CAC}))))) + (P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB}))) + ((\frac{1}{2}\cdot(P_{BCA}\cdot(P_{CAAB}\cdot P_{ABC})}{((((P_{BAAC}\cdot(P_{BAC}\cdot(P_{CBC}+(\frac{1}{2}\cdot(0+(-1\cdot P_{CBC}))))))) + (P_{BAAC}\cdot(P_{ABC}\cdot P_{ACF}))) + (-1\cdot(P_{BACC}\cdot(P_{BAC}\cdot P_{BAF})))) + (P_{ABA}\cdot(P_{BAAC}\cdot P} \tag{61}$$

$= 1,\quad$ by geometric simplifications

$$\frac{((((\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CAAB}\cdot P_{CBC}))) + ((-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{BAC}))) + (-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{CAC}))))) + (P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB}))) + ((\frac{1}{2}\cdot(P_{BCA}\cdot(P_{CAAB}\cdot P_{ABC})}{((((\frac{1}{2}\cdot(P_{BAAC}\cdot(P_{BAC}\cdot P_{CBC}))) + (P_{BAAC}\cdot(P_{ABC}\cdot P_{ACF}))) + (-1\cdot(P_{BACC}\cdot(P_{BAC}\cdot P_{BAF})))) + (P_{ABA}\cdot(P_{BAAC}\cdot P_{BACC})))} \tag{62}$$

$= 1,\quad$ by algebraic simplifications

$$\frac{((((\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CAAB}\cdot P_{CBC}))) + ((-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{BAC}))) + (-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{CAC}))))) + (P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB}))) + ((\frac{1}{2}\cdot(P_{BCA}\cdot(P_{CAAB}\cdot P_{ABC})}{((((\frac{1}{2}\cdot(P_{BAAC}\cdot(P_{BAC}\cdot P_{CBC}))) + (P_{BAAC}\cdot(P_{ABC}\cdot(P_{ACB}+(\frac{1}{2}\cdot(P_{ACC}+(-1\cdot P_{ACB}))))))) + (-1\cdot(P_{BACC}\cdot(P_{BAC}\cdot P_{BAF})))) + (P_{ABA}\cdot(P_{BAAC}} \tag{63}$$

$= 1,\quad$ by Lemma 29 (point F eliminated)

$$\frac{((((\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CAAB}\cdot P_{BCB}))) + ((-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{BAC}))) + (-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{CAC}))))) + (P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB}))) + ((\frac{1}{2}\cdot(P_{BCA}\cdot(P_{CAAB}\cdot P_{ABC})}{((((\frac{1}{2}\cdot(P_{BAAC}\cdot(P_{BAC}\cdot P_{BCB}))) + (P_{BAAC}\cdot(P_{ABC}\cdot(P_{BCA}+(\frac{1}{2}\cdot(0+(-1\cdot P_{BCA}))))))) + (-1\cdot(P_{BACC}\cdot(P_{BAC}\cdot P_{BAF})))) + (P_{ABA}\cdot(P_{BAAC}\cdot} \tag{64}$$

$= 1,\quad$ by geometric simplifications

$$\frac{((((\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CAAB}\cdot P_{BCB}))) + ((-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{BAC}))) + (-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{CAC}))))) + (P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB}))) + ((\frac{1}{2}\cdot(P_{BCA}\cdot(P_{CAAB}\cdot P_{ABC})}{((((\frac{1}{2}\cdot(P_{BAAC}\cdot(P_{BAC}\cdot P_{BCB}))) + (\frac{1}{2}\cdot(P_{BAAC}\cdot(P_{ABC}\cdot P_{BCA})))) + (-1\cdot(P_{BACC}\cdot(P_{BAC}\cdot P_{BAF})))) + (P_{ABA}\cdot(P_{BAAC}\cdot P_{BACC})))} \tag{65}$$

$= 1,\quad$ by algebraic simplifications

$$\frac{((((\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CAAB}\cdot P_{BCB})))+((-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{BAC})))+(-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{CAC})))))+(P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB})))+((\frac{1}{2}\cdot(P_{BCA}\cdot(P_{CAAB}\cdot P_{ABC})}{((((\frac{1}{2}\cdot(P_{BAAC}\cdot(P_{BAC}\cdot P_{BCB})))+(\frac{1}{2}\cdot(P_{BAAC}\cdot(P_{ABC}\cdot P_{BCA}))))+(-1\cdot(P_{BACC}\cdot(P_{BAC}\cdot(P_{BAB}+(\frac{1}{2}\cdot(P_{BAC}+(-1\cdot P_{BAB}))))))))+(P_{ABA}\cdot(P_{B}} \tag{66}$$
$= 1,$ by Lemma 29 (point F eliminated)

$$\frac{((((\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CAAB}\cdot P_{BCB})))+((-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{BAC})))+(-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{CAC})))))+(P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB})))+((\frac{1}{2}\cdot(P_{BCA}\cdot(P_{CAAB}\cdot P_{ABC})}{((((\frac{1}{2}\cdot(P_{BAAC}\cdot(P_{BAC}\cdot P_{BCB})))+(\frac{1}{2}\cdot(P_{BAAC}\cdot(P_{ABC}\cdot P_{BCA}))))+(-1\cdot(P_{BACC}\cdot(P_{BAC}\cdot(P_{BAB}+(\frac{1}{2}\cdot(P_{BAC}+(-1\cdot P_{BAB}))))))))+(P_{BAB}\cdot(P_{B}} \tag{67}$$
$= 1,$ by geometric simplifications

$$\frac{((((\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CAAB}\cdot P_{BCB})))+((-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{BAC})))+(-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CABB}\cdot P_{CAC})))))+(P_{BAC}\cdot(P_{CAAB}\cdot P_{CABB})))+((\frac{1}{2}\cdot(P_{BCA}\cdot(P_{CAAB}\cdot P_{ABC})}{((((\frac{1}{2}\cdot(P_{BAAC}\cdot(P_{BAC}\cdot P_{BCB})))+(\frac{1}{2}\cdot(P_{BAAC}\cdot(P_{ABC}\cdot P_{BCA}))))+((-\frac{1}{2}\cdot(P_{BACC}\cdot(P_{BAC}\cdot P_{BAB})))+(-\frac{1}{2}\cdot(P_{BACC}\cdot(P_{BAC}\cdot P_{BAC})))))+(P_{BAB}\cdot(P} \tag{68}$$
$= 1,$ by algebraic simplifications

$$\frac{((((\frac{1}{2}\cdot(P_{BAC}\cdot((P_{BAC}+(-1\cdot 0))\cdot P_{BCB})))+((-\frac{1}{2}\cdot(P_{BAC}\cdot((P_{BAC}+(-1\cdot P_{BAB}))\cdot P_{BAC})))+(-\frac{1}{2}\cdot(P_{BAC}\cdot((P_{BAC}+(-1\cdot P_{BAB}))\cdot P_{CAC})))))+(P_{BAC}\cdot((P_{BAC}+(}{((((\frac{1}{2}\cdot((P_{BAC}+(-1\cdot 0))\cdot(P_{BAC}\cdot P_{BCB})))+(\frac{1}{2}\cdot((P_{BAC}+(-1\cdot 0))\cdot(P_{ABC}\cdot P_{BCA}))))+((-\frac{1}{2}\cdot((P_{BAC}+(-1\cdot P_{CAC}))\cdot(P_{BAC}\cdot P_{BAB})))+} \tag{69}$$
$= 1,$ by geometric simplifications

$$\frac{(((\frac{1}{2}\cdot(P_{BAC}\cdot(P_{BAC}\cdot P_{BCB})))+(((\frac{1}{2}\cdot(P_{BAC}\cdot(P_{BAC}\cdot P_{BAC})))+(-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{BAC}\cdot P_{BAB}))))+((-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CAC}\cdot P_{BAC})))+(\frac{1}{2}\cdot(P_{BAC}\cdot(P_{CAC}\cdot P_{BAB}))))))}{(((\frac{1}{2}\cdot(P_{BAC}\cdot(P_{BAC}\cdot P_{BCB})))+(\frac{1}{2}\cdot(P_{BAC}\cdot(P_{ABC}\cdot P_{BCA}))))+(((\frac{1}{2}\cdot(P_{BAC}\cdot(P_{BAB}\cdot P_{BAC})))+(-\frac{1}{2}\cdot(P_{BAC}\cdot(P_{BAB}\cdot P_{CAC}))))+((} \tag{70}$$
$= 1,$ by algebraic simplifications

$$\frac{(((\frac{1}{2}\cdot((((BA\cdot BA)+(CA\cdot CA))+(-1\cdot(BC\cdot BC)))\cdot((((BA\cdot BA)+(CA\cdot CA))+(-1\cdot(BC\cdot BC)))\cdot(2\cdot(BC\cdot BC)))))+(((\frac{1}{2}\cdot((((BA\cdot BA)+(CA\cdot CA))+(-1\cdot(BC}{(((\frac{1}{2}\cdot((((BA\cdot BA)+(CA\cdot CA}} \tag{71}$$
$= 1,$ by geometric simplifications

$$0 = 0, \quad \text{by algebraic simplifications} \tag{72}$$

Q.E.D.

NDG conditions are:

$S_{FDE} \neq 0$ i.e., points F, D and E are not collinear (foot is not the point itself; construction based assumption)

$S_{DFF^0_{fg}} \neq S_{EFF^0_{fg}}$ i.e., lines DE and FF^0_{fg} are not parallel (construction based assumption)

Number of elimination proof steps: 26
Number of geometric proof steps: 133
Number of algebraic proof steps: 1857
Total number of proof steps: 2016
Time spent by the prover: 0.292 seconds