

GCLC Prover Output for conjecture “proof1”

Area method used

June 20, 2010

$$(1) \quad (P_{ACA} + P_{BDB}) = (2 \cdot (P_{QSQ} + P_{PRP})) \quad , \quad \text{by the statement}$$

$$(2) \quad (P_{ACA} + P_{BDB}) = ((2 \cdot P_{QSQ}) + (2 \cdot P_{PRP})) \quad , \quad \text{by algebraic simplifications}$$

$$(3) \quad (P_{ACA} + P_{BDB}) = \left(\left(2 \cdot \left(\left(P_{QDQ} + \left(\frac{1}{2} \cdot ((P_{QAQ} + (-1 \cdot P_{QDQ})) + (2 \cdot P_{DDA})) \right) \right) + \left(-\frac{1}{4} \cdot P_{DAD} \right) \right) \right) + (2 \cdot P_{PRP}) \right) \quad , \quad \text{by Lemma 33 (point } S \text{ eliminated)}$$

$$(4) \quad (P_{ACA} + P_{BDB}) = \left(\left(2 \cdot \left(\left(P_{QDQ} + \left(\frac{1}{2} \cdot ((P_{QAQ} + (-1 \cdot P_{QDQ})) + (2 \cdot 0)) \right) \right) + \left(-\frac{1}{4} \cdot P_{DAD} \right) \right) \right) + (2 \cdot P_{PRP}) \right) \quad , \quad \text{by geometric simplifications}$$

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$$(5) \quad (P_{ACA} + P_{BDB}) = \left(\left((P_{QDQ} + P_{QAQ}) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) + (2 \cdot P_{PRP}) \right) \quad , \quad \text{by algebraic simplifications}$$

$$(6) \quad (P_{ACA} + P_{BDB}) = \left(\left((P_{QDQ} + P_{QAQ}) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) + \left(2 \cdot \left(\left(P_{PCP} + \left(\frac{1}{2} \cdot ((P_{PDP} + (-1 \cdot P_{PCP})) + (2 \cdot P_{CCD})) \right) \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) \right) \right) \quad , \quad \text{by Lemma 33 (point } R \text{ eliminated)}$$

$$(7) \quad (P_{ACA} + P_{BDB}) = \left(\left((P_{QDQ} + P_{QAQ}) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) + \left(2 \cdot \left(\left(P_{PCP} + \left(\frac{1}{2} \cdot ((P_{PDP} + (-1 \cdot P_{PCP})) + (2 \cdot 0)) \right) \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) \right) \right) \quad , \quad \text{by geometric simplifications}$$

(8)

$$(P_{ACA} + P_{BDB}) = \left(\left((P_{QDQ} + P_{QAQ}) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) + \left((P_{PCP} + P_{PDP}) + \left(-\frac{1}{2} \cdot P_{CDC} \right) \right) \right), \text{ by algebraic simplifications}$$

(9)

$$(P_{ACA} + P_{BDB}) = \left(\left(\left(\left(P_{QDB} + \left(\frac{1}{2} \cdot (P_{QDC} + (-1 \cdot P_{QDB})) \right) \right) + P_{QAQ} \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) + \left((P_{PCP} + P_{PDP}) + \left(-\frac{1}{2} \cdot P_{CDC} \right) \right) \right), \text{ by Lemma 29 (point } Q \text{ eliminated)}$$

(10)

$$(P_{ACA} + P_{BDB}) = \left(\left(\left(\left(P_{BDQ} + \left(\frac{1}{2} \cdot (P_{CDQ} + (-1 \cdot P_{BDQ})) \right) \right) + P_{QAQ} \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) + \left((P_{PCP} + P_{PDP}) + \left(-\frac{1}{2} \cdot P_{CDC} \right) \right) \right), \text{ by geometric simplifications}$$

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(11)

$$(P_{ACA} + P_{BDB}) = \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDQ} \right) + \left(\frac{1}{2} \cdot P_{CDQ} \right) \right) + P_{QAQ} \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) + \left((P_{PCP} + P_{PDP}) + \left(-\frac{1}{2} \cdot P_{CDC} \right) \right) \right), \text{ by algebraic simplifications}$$

(12)

$$(P_{ACA} + P_{BDB}) = \left(\left(\left(\left(\left(\frac{1}{2} \cdot \left(P_{BDB} + \left(\frac{1}{2} \cdot (P_{BDC} + (-1 \cdot P_{BDB})) \right) \right) \right) + \left(\frac{1}{2} \cdot P_{CDQ} \right) \right) + P_{QAQ} \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) + \left((P_{PCP} + P_{PDP}) + \left(-\frac{1}{2} \cdot P_{CDC} \right) \right) \right), \text{ by Lemma 29 (point } Q \text{ eliminated)}$$

(13)

$$\left(P_{ACA} + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) = \left(\left(\left(\left(\left(\frac{1}{4} \cdot P_{BDC} \right) + \left(\frac{1}{2} \cdot P_{CDQ} \right) \right) + P_{QAQ} \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) + \left((P_{PCP} + P_{PDP}) + \left(-\frac{1}{2} \cdot P_{CDC} \right) \right) \right), \text{ by algebraic simplifications}$$

$$(14) \quad \left(P_{ACA} + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) = \left(\left(\left(\left(\left(\frac{1}{4} \cdot P_{BDC} \right) + \left(\frac{1}{2} \cdot \left(P_{CDB} + \left(\frac{1}{2} \cdot (P_{CDC} + (-1 \cdot P_{CDB})) \right) \right) \right) \right) \right) + P_{QAQ} \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) \\ + \left((P_{PCP} + P_{PDP}) + \left(-\frac{1}{2} \cdot P_{CDC} \right) \right) \quad , \quad \text{by Lemma 29 (point } Q \text{ eliminated)}$$

$$(15) \quad \left(P_{ACA} + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) = \left(\left(\left(\left(\left(\frac{1}{4} \cdot P_{BDC} \right) + \left(\frac{1}{2} \cdot \left(P_{BDC} + \left(\frac{1}{2} \cdot (P_{CDC} + (-1 \cdot P_{BDC})) \right) \right) \right) \right) \right) + P_{QAQ} \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) \\ + \left((P_{PCP} + P_{PDP}) + \left(-\frac{1}{2} \cdot P_{CDC} \right) \right) \quad , \quad \text{by geometric simplifications}$$

$$(16) \quad \left(P_{ACA} + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) = \left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + P_{QAQ} \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) \\ + (P_{PCP} + P_{PDP}) \quad , \quad \text{by algebraic simplifications}$$

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$$(17) \quad \left(P_{ACA} + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) = \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(P_{QAB} + \left(\frac{1}{2} \cdot (P_{QAC} + (-1 \cdot P_{QAB})) \right) \right) \right) \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) \\ + (P_{PCP} + P_{PDP}) \quad , \quad \text{by Lemma 29 (point } Q \text{ eliminated)}$$

$$(18) \quad \left(P_{ACA} + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) = \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(P_{BAQ} + \left(\frac{1}{2} \cdot (P_{CAQ} + (-1 \cdot P_{BAQ})) \right) \right) \right) \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) \\ + (P_{PCP} + P_{PDP}) \quad , \quad \text{by geometric simplifications}$$

$$(19) \quad \left(P_{ACA} + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) = \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\frac{1}{2} \cdot P_{BAQ} \right) + \left(\frac{1}{2} \cdot P_{CAQ} \right) \right) \right) \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) \\ + (P_{PCP} + P_{PDP}) \quad , \quad \text{by algebraic simplifications}$$

$$\begin{aligned}
(20) \quad \left(P_{ACA} + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) &= \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\frac{1}{2} \cdot \left(P_{BAB} + \left(\frac{1}{2} \cdot (P_{BAC} + (-1 \cdot P_{BAB})) \right) \right) \right) + \left(\frac{1}{2} \cdot P_{CAQ} \right) \right) \right) \right) \right. \\
&\quad \left. + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) + (P_{PCP} + P_{PDP}) \quad , \quad \text{by Lemma 29 (point } Q \text{ eliminated)}
\end{aligned}$$

$$\begin{aligned}
(21) \quad \left(P_{ACA} + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) &= \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{4} \cdot P_{BAC} \right) \right) + \left(\frac{1}{2} \cdot P_{CAQ} \right) \right) \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) \right. \\
&\quad \left. + (P_{PCP} + P_{PDP}) \right) \quad , \quad \text{by algebraic simplifications}
\end{aligned}$$

$$\begin{aligned}
(22) \quad \left(P_{ACA} + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) &= \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{4} \cdot P_{BAC} \right) \right) + \left(\frac{1}{2} \cdot \left(P_{CAB} + \left(\frac{1}{2} \cdot (P_{CAC} + (-1 \cdot P_{CAB})) \right) \right) \right) \right) \right) \right) \right. \\
&\quad \left. + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) + (P_{PCP} + P_{PDP}) \quad , \quad \text{by Lemma 29 (point } Q \text{ eliminated)}
\end{aligned}$$

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$$\begin{aligned}
(23) \quad \left(P_{ACA} + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) &= \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{4} \cdot P_{BAC} \right) \right) + \left(\frac{1}{2} \cdot \left(P_{BAC} + \left(\frac{1}{2} \cdot (P_{ACA} + (-1 \cdot P_{BAC})) \right) \right) \right) \right) \right) \right) \right. \\
&\quad \left. + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) + (P_{PCP} + P_{PDP}) \quad , \quad \text{by geometric simplifications}
\end{aligned}$$

$$\begin{aligned}
(24) \quad \left(\left(\frac{3}{4} \cdot P_{ACA} \right) + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) &= \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{2} \cdot P_{BAC} \right) \right) \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) \right. \\
&\quad \left. + (P_{PCP} + P_{PDP}) \right) \quad , \quad \text{by algebraic simplifications}
\end{aligned}$$

$$\begin{aligned}
(25) \quad \left(\left(\frac{3}{4} \cdot P_{ACA} \right) + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) &= \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{2} \cdot P_{BAC} \right) \right) \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) \right. \\
&\quad \left. + \left(\left(P_{PCA} + \left(\frac{1}{2} \cdot (P_{PCB} + (-1 \cdot P_{PCA})) \right) \right) + P_{PDP} \right) \right) \quad , \quad \text{by Lemma 29 (point } P \text{ eliminated)}
\end{aligned}$$

$$\begin{aligned}
(26) \quad \left(\left(\frac{3}{4} \cdot P_{ACA} \right) + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) &= \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{2} \cdot P_{BAC} \right) \right) \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) \right. \\
&\quad \left. + \left(\left(P_{ACP} + \left(\frac{1}{2} \cdot (P_{BCP} + (-1 \cdot P_{ACP})) \right) \right) + P_{PDP} \right) \right) \quad , \quad \text{by geometric simplifications}
\end{aligned}$$

$$\begin{aligned}
(27) \quad \left(\left(\frac{3}{4} \cdot P_{ACA} \right) + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) &= \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{2} \cdot P_{BAC} \right) \right) \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) \right. \\
&\quad \left. + \left(\left(\left(\frac{1}{2} \cdot P_{ACP} \right) + \left(\frac{1}{2} \cdot P_{BCP} \right) \right) + P_{PDP} \right) \right) \quad , \quad \text{by algebraic simplifications}
\end{aligned}$$

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$$\begin{aligned}
(28) \quad \left(\left(\frac{3}{4} \cdot P_{ACA} \right) + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) &= \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{2} \cdot P_{BAC} \right) \right) \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) \right. \\
&\quad \left. + \left(\left(\left(\frac{1}{2} \cdot \left(P_{ACA} + \left(\frac{1}{2} \cdot (P_{ACB} + (-1 \cdot P_{ACA})) \right) \right) \right) + \left(\frac{1}{2} \cdot P_{BCP} \right) \right) \right) \right. \\
&\quad \left. + P_{PDP} \right) \quad , \quad \text{by Lemma 29 (point } P \text{ eliminated)}
\end{aligned}$$

$$\begin{aligned}
(29) \quad \left(\left(\frac{1}{2} \cdot P_{ACA} \right) + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) &= \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{2} \cdot P_{BAC} \right) \right) \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) \right. \\
&\quad \left. + \left(\left(\left(\frac{1}{4} \cdot P_{ACB} \right) + \left(\frac{1}{2} \cdot P_{BCP} \right) \right) + P_{PDP} \right) \right) \quad , \quad \text{by algebraic simplifications}
\end{aligned}$$

$$\begin{aligned}
(30) \quad \left(\left(\frac{1}{2} \cdot P_{ACA} \right) + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) &= \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{2} \cdot P_{BAC} \right) \right) \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) \right. \\
&\quad \left. + \left(\left(\left(\frac{1}{4} \cdot P_{ACB} \right) + \left(\frac{1}{2} \cdot \left(P_{BCA} + \left(\frac{1}{2} \cdot (P_{BCB} + (-1 \cdot P_{BCA})) \right) \right) \right) \right) \right) \right. \\
&\quad \left. + P_{DDP} \right) \quad , \quad \text{by Lemma 29 (point } P \text{ eliminated)}
\end{aligned}$$

$$\begin{aligned}
(31) \quad \left(\left(\frac{1}{2} \cdot P_{ACA} \right) + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) &= \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{2} \cdot P_{BAC} \right) \right) \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) \right. \\
&\quad \left. + \left(\left(\left(\frac{1}{4} \cdot P_{ACB} \right) + \left(\frac{1}{2} \cdot \left(P_{ACB} + \left(\frac{1}{2} \cdot (P_{BCB} + (-1 \cdot P_{ACB})) \right) \right) \right) \right) \right) \right. \\
&\quad \left. + P_{DDP} \right) \quad , \quad \text{by geometric simplifications}
\end{aligned}$$

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$$\begin{aligned}
(32) \quad \left(\left(\frac{1}{2} \cdot P_{ACA} \right) + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) &= \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{2} \cdot P_{BAC} \right) \right) \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) \right. \\
&\quad \left. + \left(\left(\left(\frac{1}{2} \cdot P_{ACB} \right) + \left(\frac{1}{4} \cdot P_{BCB} \right) \right) + P_{DDP} \right) \right) \quad , \quad \text{by algebraic simplifications}
\end{aligned}$$

$$\begin{aligned}
(33) \quad \left(\left(\frac{1}{2} \cdot P_{ACA} \right) + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) &= \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{2} \cdot P_{BAC} \right) \right) \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) \right. \\
&\quad \left. + \left(\left(\left(\frac{1}{2} \cdot P_{ACB} \right) + \left(\frac{1}{4} \cdot P_{BCB} \right) \right) \right) \right. \\
&\quad \left. + \left(P_{DDA} + \left(\frac{1}{2} \cdot (P_{DDB} + (-1 \cdot P_{DDA})) \right) \right) \right) \quad , \quad \text{by Lemma 29 (point } P \text{ eliminated)}
\end{aligned}$$

$$\begin{aligned}
(34) \quad \left(\left(\frac{1}{2} \cdot P_{ACA} \right) + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) &= \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{2} \cdot P_{BAC} \right) \right) \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) \right. \\
&\quad \left. + \left(\left(\left(\frac{1}{2} \cdot P_{ACB} \right) + \left(\frac{1}{4} \cdot P_{BCB} \right) \right) \right) \right. \\
&\quad \left. + \left(P_{ADP} + \left(\frac{1}{2} \cdot (P_{BDP} + (-1 \cdot P_{ADP})) \right) \right) \right) \quad , \quad \text{by geometric simplifications}
\end{aligned}$$

$$\begin{aligned}
(35) \quad \left(\left(\frac{1}{2} \cdot P_{ACA} \right) + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) &= \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{2} \cdot P_{BAC} \right) \right) \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) \right. \\
&\quad \left. + \left(\left(\left(\frac{1}{2} \cdot P_{ACB} \right) + \left(\frac{1}{4} \cdot P_{BCB} \right) \right) \right) \right. \\
&\quad \left. + \left(\left(\frac{1}{2} \cdot P_{ADP} \right) + \left(\frac{1}{2} \cdot P_{BDP} \right) \right) \right) \quad , \quad \text{by algebraic simplifications}
\end{aligned}$$

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$$\begin{aligned}
(36) \quad &\left(\left(\frac{1}{2} \cdot P_{ACA} \right) \right. \\
&\quad \left. + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) = \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{2} \cdot P_{BAC} \right) \right) \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) + \left(\left(\left(\frac{1}{2} \cdot P_{ACB} \right) + \left(\frac{1}{4} \cdot P_{BCB} \right) \right) \right) \right. \\
&\quad \left. + \left(\left(\frac{1}{2} \cdot \left(P_{ADA} + \left(\frac{1}{2} \cdot (P_{ADB} + (-1 \cdot P_{ADA})) \right) \right) \right) + \left(\frac{1}{2} \cdot P_{BDP} \right) \right) \right) \quad , \quad \text{by Lemma 29 (point } P \text{ eliminated)}
\end{aligned}$$

$$\begin{aligned}
(37) \quad &\left(\left(\frac{1}{2} \cdot P_{ACA} \right) \right. \\
&\quad \left. + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) = \left(\left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{2} \cdot P_{BAC} \right) \right) \right) + \left(-\frac{1}{2} \cdot P_{DAD} \right) \right) + \left(\left(\left(\frac{1}{2} \cdot P_{ACB} \right) + \left(\frac{1}{4} \cdot P_{BCB} \right) \right) \right) \right. \\
&\quad \left. + \left(\left(\frac{1}{2} \cdot \left(P_{DAD} + \left(\frac{1}{2} \cdot (P_{ADB} + (-1 \cdot P_{DAD})) \right) \right) \right) + \left(\frac{1}{2} \cdot P_{BDP} \right) \right) \right) \quad , \quad \text{by geometric simplifications}
\end{aligned}$$

$$(38) \quad \left(\left(\frac{1}{2} \cdot P_{ACA} \right) + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) = \left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{2} \cdot P_{BAC} \right) \right) \right) + \left(\left(\left(\frac{1}{2} \cdot P_{ACB} \right) + \left(\frac{1}{4} \cdot P_{BCB} \right) \right) \right. \right. \\ \left. \left. + \left(\left(\left(\frac{1}{4} \cdot P_{ADB} \right) + \left(-\frac{1}{4} \cdot P_{DAD} \right) \right) + \left(\frac{1}{2} \cdot P_{BDP} \right) \right) \right) \right), \text{ by algebraic simplifications}$$

$$(39) \quad \left(\left(\frac{1}{2} \cdot P_{ACA} \right) + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) = \left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{2} \cdot P_{BAC} \right) \right) \right) + \left(\left(\left(\frac{1}{2} \cdot P_{ACB} \right) + \left(\frac{1}{4} \cdot P_{BCB} \right) \right) \right. \right. \\ \left. \left. + \left(\left(\left(\frac{1}{4} \cdot P_{ADB} \right) + \left(-\frac{1}{4} \cdot P_{DAD} \right) \right) + \left(\frac{1}{2} \cdot \left(P_{BDA} + \left(\frac{1}{2} \cdot (P_{BDB} + (-1 \cdot P_{BDA})) \right) \right) \right) \right) \right) \right), \text{ by Lemma 29 (point } P \text{ eliminated)}$$

$$(40) \quad \left(\left(\frac{1}{2} \cdot P_{ACA} \right) + \left(\frac{3}{4} \cdot P_{BDB} \right) \right) = \left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{2} \cdot P_{BAC} \right) \right) \right) + \left(\left(\left(\frac{1}{2} \cdot P_{ACB} \right) + \left(\frac{1}{4} \cdot P_{BCB} \right) \right) \right. \right. \\ \left. \left. + \left(\left(\left(\frac{1}{4} \cdot P_{ADB} \right) + \left(-\frac{1}{4} \cdot P_{ADA} \right) \right) + \left(\frac{1}{2} \cdot \left(P_{ADB} + \left(\frac{1}{2} \cdot (P_{BDB} + (-1 \cdot P_{ADB})) \right) \right) \right) \right) \right) \right), \text{ by geometric simplifications}$$

$$(41) \quad \left(\left(\frac{1}{2} \cdot P_{ACA} \right) + \left(\frac{1}{2} \cdot P_{BDB} \right) \right) = \left(\left(\left(\left(\frac{1}{2} \cdot P_{BDC} \right) + \left(-\frac{1}{4} \cdot P_{CDC} \right) \right) + \left(\left(\frac{1}{4} \cdot P_{BAB} \right) + \left(\frac{1}{2} \cdot P_{BAC} \right) \right) \right) + \left(\left(\left(\frac{1}{2} \cdot P_{ACB} \right) + \left(\frac{1}{4} \cdot P_{BCB} \right) \right) \right. \right. \\ \left. \left. + \left(\left(\frac{1}{2} \cdot P_{ADB} \right) + \left(-\frac{1}{4} \cdot P_{ADA} \right) \right) \right) \right), \text{ by algebraic simplifications}$$

$$\begin{aligned}
& \left(\left(\frac{1}{2} \cdot (2 \cdot (AC \cdot AC)) \right) + \left(\frac{1}{2} \cdot (2 \cdot (BD \cdot BD)) \right) \right) \\
& = \left(\left(\left(\left(\frac{1}{2} \cdot (((BD \cdot BD) + (CD \cdot CD)) + (-1 \cdot (BC \cdot BC))) \right) + \left(-\frac{1}{4} \cdot (2 \cdot (CD \cdot CD)) \right) \right) \right) \right. \\
(42) \quad & \quad \quad \quad + \left(\left(\frac{1}{4} \cdot (2 \cdot (BA \cdot BA)) \right) + \left(\frac{1}{2} \cdot (((BA \cdot BA) + (AC \cdot AC)) + (-1 \cdot (BC \cdot BC))) \right) \right) \Big) \\
& \quad \quad \quad + \left(\left(\left(\frac{1}{2} \cdot (((AC \cdot AC) + (BC \cdot BC)) + (-1 \cdot (BA \cdot BA))) \right) + \left(\frac{1}{4} \cdot (2 \cdot (BC \cdot BC)) \right) \right) \right) \\
& \quad \quad \quad + \left(\left(\left(\frac{1}{2} \cdot (((AD \cdot AD) + (BD \cdot BD)) + (-1 \cdot (BA \cdot BA))) \right) + \left(-\frac{1}{4} \cdot (2 \cdot (AD \cdot AD)) \right) \right) \right) \Big) \quad , \quad \text{by geometric simplifications}
\end{aligned}$$

$$(43) \quad \quad \quad 0 = 0 \quad \quad \quad , \quad \text{by algebraic simplifications}$$

Q.E.D.

There are no ndg conditions.

Number of elimination proof steps: 14

Number of geometric proof steps: 42

Number of algebraic proof steps: 160

Total number of proof steps: 216

Time spent by the prover: 0.008 seconds